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# A Numerical Study of Characteristic Slow-Transient Behavior of a Compressible 2D Gas-Liquid Two-Fluid Model

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**Abstract.** The purpose of this paper is to gain some insight into the characteristic behavior of a general compressible two-fluid gas-liquid model in 2D by using numerical computations. Main focus is on mass transport phenomena. Relatively few numerical results in higher dimensions can be found in the literature for this two-fluid model, in particular, for cases where mass transport dynamics are essential. We focus on natural extensions to 2D of known 1D benchmark test cases, like water faucet and gas-liquid separation, previously employed by many researchers for the purpose of testing various numerical schemes. For the numerical investigations, the WIMF discretization method introduced in [SIAM J. Sci. Comput. 26 (2005), 1449] is applied, in combination with a standard dimensional splitting approach. Highly complicated flow patterns are observed reflecting the balance between acceleration forces, gravity, interfacial forces, and pressure gradients. An essential ingredient in these results is the appearance of single-phase regions in combination with mixture regions (dispersed flow). Solutions are calculated and shown from early times until a steady state is reached. Grid refinement studies are included to demonstrate that the obtained solutions are not grid-sensitive.

**AMS subject classifications**: 76T10, 76N10, 65M12, 35L65

**Key words**: two-fluid model, hyperbolic, numerical scheme, pressure-velocity coupling, single-phase flow.

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# 1 Introduction

The starting point for this paper is a general, isothermal compressible two-fluid model (2D variant) in the following form where the index l,g refers to a liquid and gas phase, respectively:

$$\frac{\partial(\alpha_g\rho_g)}{\partial t} + \nabla\cdot(\alpha_g\rho_g\mathbf{v}_g) = 0, \tag{1.1a}$$

$$\frac{\partial(\alpha_l\rho_l)}{\partial t} + \nabla\cdot(\alpha_l\rho_l\mathbf{v}_l) = 0, \tag{1.1b}$$

$$\frac{\partial(\alpha_g\rho_g\mathbf{v}_g)}{\partial t} + \nabla\cdot(\alpha_g\rho_g\mathbf{v}_g\otimes\mathbf{v}_g) + \alpha_g\nabla p + \Delta p\nabla\alpha_g = \mathbf{Q}_g + \mathbf{M}_g, \tag{1.1c}$$

$$\frac{\partial(\alpha_l\rho_l\mathbf{v}_l)}{\partial t} + \nabla\cdot(\alpha_l\rho_l\mathbf{v}_l\otimes\mathbf{v}_l) + \alpha_l\nabla p + \Delta p\nabla\alpha_l = \mathbf{Q}_l + \mathbf{M}_l. \tag{1.1d}$$

Here $\alpha_g, \alpha_l$ are the volume fractions which satisfy the relation

$$\alpha_l + \alpha_g = 1.$$

Furthermore, $\rho_l(p), \rho_g(p)$ are fluid densities, $p$ is pressure, $\mathbf{v}_l, \mathbf{v}_g$ are fluid velocities, $\mathbf{Q}_l, \mathbf{Q}_g$ represent external forces (friction and gravity), $\mathbf{M}_l, \mathbf{M}_g$ represent interfacial forces modelling interactions between the two phases. In particular,

$$\mathbf{M}_l + \mathbf{M}_g = 0.$$

The model must be supplemented with equations of state for the gas and liquid phase. Moreover, expressions must be given for the interphase drag force, typically in the form

$$\mathbf{M}_l = -\mathbf{M}_g = C(\alpha_g, \rho_l, \rho_g)|\mathbf{v}_g - \mathbf{v}_l|(\mathbf{v}_g - \mathbf{v}_l),$$

see for example [11]. Similarly, expressions for $\mathbf{Q}_g, \mathbf{Q}_l$ must be specified. The $\Delta p$ term is required in order to make the model well-defined. Several expressions for $\Delta p$ have been proposed in the literature. In this work the purpose of the interface correction term $\Delta p$ is to ensure that the model becomes hyperbolic (real eigenvalues), and may have little physical justification. We refer to [13] and references therein for more details.

**Different discretization approaches**

As described in the recent book edited by Prosperetti and Tryggvason [31] there are, roughly speaking, two different classes of discretization techniques that have been used for solving the two-fluid model. The first one is referred to as *segregated* methods, the second type as *coupled* methods. The distinction between these two is not very clear, nevertheless, by relating the various schemes to these two different groups it may be easier to identify similarities and differences.