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# A Truly Boundary-Only Meshfree Method Applied to Kirchhoff Plate Bending Problems

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**Abstract.** The boundary particle method (BPM) is a truly boundary-only collocation scheme, whose basis function is the high-order nonsingular general solution or singular fundamental solution, based on the recursive composite multiple reciprocity method (RC-MRM). The RC-MRM employs the high-order composite differential operator to solve a much wider variety of inhomogeneous problems with boundary-only collocation nodes while significantly reducing computational cost via a recursive algorithm. In this study, we simulate the Kirchhoff plate bending problems by the BPM based on the RC-MRM. Numerical results show that this approach produces accurate solutions of plates subjected to various loadings with boundary-only discretization.

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**Key words**: Boundary particle method, recursive composite multiple reciprocity method, Kirchhoff plate, boundary-only, meshfree.

## 1 Introduction

In recent decades, the boundary-type meshfree methods, such as method of fundamental solution (MFS) [1–3], boundary knot method (BKM) [4], boundary collocation method (BCM) [5], regularized meshless method (RMM) [6, 7] and boundary node method (BNM) [8, 9], have attracted a lot of attention in the numerical solution of various partial differential equations. All the above-mentioned boundary methods can solve homogeneous problems with boundary-only discretization. However, these methods require inner nodes in conjunction with the other techniques to handle inhomogeneous problems, such as quasi-Monte-Carlo method [10], dual reciprocity method (DRM) [11] and multiple reciprocity method (MRM) [12].

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Since 1980s the DRM and MRM have been emerging as the two most promising techniques to handle inhomogeneous problems in conjunction with the boundary type methods [11–13]. The striking advantage of the MRM over the DRM is that it does not require using inner nodes at all for the particular solution. To take advantage of truly boundary-only merit of the MRM, Chen [14, 15] developed the MRM-based meshfree boundary particle method (BPM). However, the standard MRM is computationally expensive in the construction of the interpolation matrix and has limited feasibility for general inhomogeneous problems due to its use of high-order Laplacian operators in the annihilation process [12]. Chen and Jin [16, 17] presented the recursive composite multiple reciprocity method (RC-MRM), which employs the high-order composite differential operators to vanish the inhomogeneous term of various types. The RC-MRM significantly expands the application territory of the BPM to a much wider variety of inhomogeneous problems. In addition, the RC-MRM includes a recursive algorithm to dramatically reduce the total computing cost.

This paper is organized as follows. Section 2 introduces the BPM based on RC-MRM through its discretization to the Kirchhoff plate bending problems. The efficiency and utility of this new technique are numerically examined in Section 3. Section 4 concludes this paper with some remarks and opening issues.

## 2 RC-MRM based BPM for plate bending

Without lose of generality, this section introduces the BPM through its discretization to the Kirchhoff plate problems.

### 2.1 Plate bending

The deflection of a thin plate under a distributed loading is governed by the governing equation

$$\nabla^4 w = \frac{q}{D}, \tag{2.1}$$

where $w$ is the deflection of the middle surface of plate, $\nabla^4$ denotes the biharmonic operator, and $D=Eh^3/\big(12(1-\nu^2)\big)$ represents the flexural rigidity.

At every boundary point, the two boundary conditions have to be satisfied, which are a combination of the following conditions: displacement, normal slope, bending moment, and effective shear force. In this study, the following three types of boundary conditions are encountered: (1) Clamped edge, denoted by $C$ in this paper: $w=0$, $\theta_n=0$, where $w$ and $\theta_n$ denote the displacement and normal slope condition, respectively. (2) Simply supported edge, denoted by $S$ in this paper: $w=0$, $M_n=0$, where $M_n$ represents the bending moment condition. (3) Free edge, denoted by $F$ in this paper: $M_n=0$, $V_n=0$, where $V_n$ expresses the effective shear force.

The above boundary conditions can be expressed in terms of the deflection $w$ as follows.