Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 3, pp. 402-414 (2009)

June 2009

Adaptive Optimal Control of the Flapping Rule of a Fixed Flapping Plate

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Received 29 December 2008; Accepted (in revised version) 11 February 2009

Available online 22 April 2009

Abstract. In this paper, with the use of the moving boundary computational fluid dynamics method, we developed a new real-time optimal control method which can be used to find the optimal flapping mode of a fixed flapping plate. The results show that there is a 54.0% increase in the thrust obtained by the unsteady optimal flapping rule. In addition, to reduce the cost of computation and to have a better understanding of the flapping rule, the maximum velocity at the end tip of the flapping plate is taken as the objective functional, with which the thrust is increased by 22.9%.

AMS subject classifications: 76D55

Key words: Flapping rule, optimal control, moving boundary CFD method.

1 Introduction

The Nature creates millions of strange creatures during the billions years of evolution, and every day these living creatures move around the world in their graceful, unique and the most energy saving way. But up to now, very little has been known about the mechanism of fluid mechanics of various unsteady boundary motions, such as the moving body surface, associated with the locomotion of these creatures. We hope to better understand the inscrutability of animal motion by means of studying the unsteady optimal control of the adaptive smart surface in complex flows.

On the other hand, in the community of fluid mechanics, the techniques of unsteady control and flow control with compliant surface are attracting researchers attentions. The rapid development of MEMS, MAFC and smart materials, such as the

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shape memory alloy, makes the dream of control of fluid motion with an optimal surface to come true.

As the first step, we study the optimal flapping rule of a flapping plate, to find the optimal motion mode. A new real-time optimal control method, which is applied to adaptively control of the flapping rule of a fixed flapping plate, is developed. In addition, in order to reduce the cost of computation and deepen the understanding of the optimal flapping rule, we take the maximum velocity at the end tip of the plate as the objective functional and optimize the rule directly. A relevant study of self-propelled swimming of a fish and fish school can be found in [1].

2 Numerical method and the algorithm of optimal control

2.1 Numerical algorithm and code verifications

We used the finite-volume method provided by Ferziger & Peric [2] to solve the two-dimensional version of the incompressible Navier-Stokes and continuity equations, in the following Cartesian-component (i=1, 2) integral form,

$$\frac{\partial}{\partial t}\int_{\Omega}\rho u_i \mathrm{d}\Omega + \int_S \rho u_i u_n \mathrm{d}S = \int_S \tau_{ij} n_j \mathrm{d}S - \int_S p n_i \mathrm{d}S, \tag{2.1}$$

$$\int_S \rho u_n \mathrm{d}S = 0, \tag{2.2}$$

where τ_{ij} is the viscous stress tensor. A second-order implicit three-time-level scheme was used for integration in time. The surface integral in (2.1) is split into four control volume (CV) face integrals approximated by the midpoint rule. As a result, the spatial precision of the algorithm is of second order.

When the cell faces move, the conservation of mass (and all other conserved quantities) is not necessarily ensured if the grid velocities are used to calculate the mass fluxes. Mass conservation can be obtained by enforcing the so-called *space conservation law*, which can be thought of as the continuity equation for zero fluid velocity:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega} \mathrm{d}\Omega - \int_S \vec{u}_b \cdot \vec{n}\ \mathrm{d}S = 0, \tag{2.3}$$

where $\vec{u}_b$ is the velocity of CV cell. This equation describes the conservation of space when the CV changes its shape and/or position with time. In discretized form, (2.3) reads

$$\frac{(\Delta\Omega)^{n+1} - (\Delta\Omega)^n}{\Delta t} = \sum_c (\vec{u}_b \cdot \vec{n})_c S_c, \qquad c = e,\ w,\ n,\ s, \tag{2.4}$$

where e, w, n, and s stand for the right, left, top and bottom faces of the cell, respectively. For the implicit Euler scheme, the discretized continuity equation becomes

$$\frac{(\rho\Delta\Omega)^{n+1} - (\rho\Delta\Omega)^n}{\Delta t} + \sum_c \dot{m}_c = 0\,, \qquad c = e,\ w,\ n,\ s, \tag{2.5}$$