Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 4, pp. 451-480

DOI: 10.4208/aamm.09-m0913
August 2009

A Parameter–Free Generalized Moment Limiter for High-Order Methods on Unstructured Grids

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Received 09 January 2009; Accepted (in revised version) 07 March 2009
Available online 18 June 2009

Abstract. A parameter-free limiting technique is developed for high-order unstructured-grid methods to capture discontinuities when solving hyperbolic conservation laws. The technique is based on a "troubled-cell" approach, in which cells requiring limiting are first marked, and then a limiter is applied to these marked cells. A parameter-free accuracy-preserving TVD marker based on the cell-averaged solutions and solution derivatives in a local stencil is compared to several other markers in the literature in identifying "troubled cells". This marker is shown to be reliable and efficient to consistently mark the discontinuities. Then a compact high-order hierarchical moment limiter is developed for arbitrary unstructured grids. The limiter preserves a degree p polynomial on an arbitrary mesh. As a result, the solution accuracy near smooth local extrema is preserved. Numerical results for the high-order spectral difference methods are provided to illustrate the accuracy, effectiveness, and robustness of the present limiting technique.

AMS subject classifications: 65M70, 76M20, 76M22
Key words: Limiter, shock-capturing, high-order, unstructured grids.

1 Introduction

A nonlinear hyperbolic conservation law can generate discontinuities even if the initial solution is smooth. A significant computational challenge with a nonlinear hyperbolic conservation law is the resolution of such discontinuities, which has been a very active area of research for over four decades. However, any linear scheme higher than first order accuracy cannot generate monotonic solutions, according to the Godunov theorem [8]. This means linear schemes of 2nd-order and higher will produce spurious oscillations near discontinuities due to the so-called Gibbs phenomenon, which can result in numerical instability and non-physical data, such as negative pressure

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or density. Early research work on shock-capturing relied on numerical diffusion to smear the discontinuities so that they can be captured as part of the numerical solution [14, 20, 25, 40]. Besides the existence of user-defined parameters, the historical drawback of the artificial viscosity approach is that the added terms are frequently too dissipative in certain regions of the flow. Later, another type of approach was developed based on flux limiting, which introduced numerical diffusion implicitly to reduce or remove spurious oscillations. Pioneering works in flux limiting include the FCT [3], the MUSCL and related methods [9, 38, 39], and TVD methods [10, 44]. However, the flux-limiting and TVD methods suffered from accuracy-degradation to first-order at local extrema in smooth regions.

High-order (3^{rd}-order and higher) shock-capturing algorithms have the potential to obtain sharp non-oscillatory shock profile and simultaneously preserve accuracy in smooth regions. The challenge of producing oscillation-free numerical solutions is tougher for high-order methods than for lower order ones because of much reduced numerical dissipation. The artificial viscosity method has been improved [6, 7, 36] to minimize undesirable dissipation by using a spectrally vanishing viscosity approach based on high-order derivatives of the strain rate tensor, though there still exist user-defined parameters that can be mesh or problem dependent. The ENO [9] and WENO methods [15] used the idea of adaptive stencils in the reconstruction procedure based on the smoothness of the local numerical solution. However, due to a lack of compactness, the implementation of both ENO and WENO methods is complicated on arbitrary unstructured meshes, especially for 3D problems.

High-order methods designed for unstructured meshes offer obvious advantages in geometric flexibility. Examples of such methods include the discontinuous Galerkin (DG) method [4, 5, 30], the multi-domain staggered- grid method [16, 17], the spectral volume method [41, 42], the spectral difference (SD) method [22, 34]. A review of these and other unstructured-grid based high-order methods can be found in [43]. These high-order methods are usually compact, meaning cells are coupled with their immediate face neighbors. Compact high-order methods are much more suitable for massively parallel machines as the amount of data communication is minimized. In designing limiters for such methods, it is natural to require that the limiters should be compact. There have been many notable developments in limiters for high-order methods in the last decade. Many of the limiters employ the so-called "troubled cell" (TC) approach, in which "oscillatory" cells are marked first, and the solutions in these cells are re-generated to remove or reduce the oscillations satisfying certain criteria such as mean-preserving. The idea is first developed in [5], and then further extended in [2]. In [5], a limiter developed for the finite volume method [1] was used. The moment limiter developed in [2] can be viewed as the generalization of the minmod limiter [39] to higher order derivatives or moments. The central DG scheme proposed in [23] is a further generalization of the MUSCL scheme and the moment limiter. Other more recent developments include the use of WENO [28] and Hermite WENO [24, 29] schemes to generate the reconstruction in "troubled cells". High-order limiters based on artificial viscosity have also been investigated by various researchers [13,26]. In the