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Heat Convection Between Two Confocal Elliptic Tubes Placed at Different Orientations

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Abstract. In this paper, transient and steady natural convection heat transfer in an elliptical annulus has been investigated. The annulus occupies the space between two horizontal concentric tubes of elliptic cross-section. The resulting velocity and thermal fields are predicted at different annulus orientations assuming isothermal surfaces. The full governing equations of mass, momentum and energy are solved numerically using the Fourier Spectral method. The heat convection process between the two tubes depends on Rayleigh number, Prandtl number, angle of inclination of tube axes and the geometry and dimensions of both tubes. The Prandtl number and inner tube axis ratio are fixed at 0.7 and 0.5, respectively. The problem is solved for the two Rayleigh numbers of 10^4 and 10^5 considering a ratio between the two major axes up to 3 while the angle of orientation of the minor axes varies from 0 to 90°. The results for local and average Nusselt numbers are obtained and discussed together with the details of both flow and thermal fields. For isothermal heating conditions, the study has shown an optimum value for major axes ratio that minimizes the rate of heat transfer between the two tubes. Another important aspect of this paper is to prove the successful use of the Fourier Spectral Method in solving confined flow and heat convection problems.

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1 Introduction

As it has been the case for fundamental engineering problems, natural convection in an annular enclosure has been the target of persistent scientific research. The research

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has been directed to this problem because of its pertinence to many practical engineering applications. These applications include nuclear reactor systems, thermal storage and solar heating systems. The annular space can be geometrically formed by the region between either concentric or eccentric two elliptical tubes. The concentric elliptical tube geometry can represent different annuli configurations ranging from the annulus formed between two concentric circular tubes to annulus formed by a flat plate surrounded by an elliptical tube. In case of annulus formed between two concentric, confocal elliptic tubes, the geometry of the annulus is controlled by two parameters. These are the ratio of the major axis of outer tube to the major axis of inner tube and the axis ratio of both tubes.

The problem of steady natural convection in a horizontal annulus between two concentric/eccentric circular tubes was considered by many researchers. The work of Kuehn and Goldstein [1] comes in the vanguard, with pioneering experimental and theoretical results and with good, thorough literature review. The results in that work covered the most details of steady heat transfer characteristics and flow patterns developed in the annulus. The heat transfer results were presented in terms of Nusselt number and equivalent conductivity. The equivalent conductivity is defined as the actual heat flux divided by the heat flux that would occur by pure conduction. The authors have further extended their experimental study [2] to include the natural convection in an eccentric circular annulus; the case which has been considered later by a number of researchers [3–10].

In comparison with the circular concentric and eccentric cases, a few number of studies were conducted on natural convection in a non-circular enclosures such as the elliptic annulus considered in this study. Lee and Lee [11] have investigated experimentally and theoretically, through a few test cases, steady natural convection of air in a symmetric annulus formed between two concentric, confocal elliptic tubes. They used the conventional finite-difference method to solve the governing equations that were written in stream function-vorticity formulation. Their numerical results were verified through comparisons with the experimental results performed using the Mach-Zehnder interferometer and smoke flow visualization. Elshamy et al. [12] used the finite-volume approach to investigate numerically the steady natural convection in horizontal symmetric annulus formed between two confocal elliptic tubes. They presented the results in terms of the local and average Nusselt numbers together with correlations for the average Nusselt number.

In their numerical investigation to study the steady natural convection in some horizontal concentric and eccentric elliptical configurations, Cheng and Chao [13] used the finite-volume discretization technique method to solve the governing equations that were written in terms of primitive variables. They discretized the governing equations using a curvilinear grid system that was generated by a body-fitted coordinate transformation method. They presented their heat transfer results for a number of different symmetrical elliptic configurations, expressing their results in terms of equivalent conductivity.

Zhu et al. [14] used the Differential Quadrature (DQ) method to study numerically