Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **1**, No. 5, pp. 683-698

DOI: 10.4208/aamm.09-m0925
October 2009

Iterative Method for Solving a Problem with Mixed Boundary Conditions for Biharmonic Equation

Dang Quang A[1,*] and Le Tung Son[2]

[1] *Institute of Information Technology 18 Hoang Quoc Viet, Cau giay, Hanoi, Vietnam*

[2] *Pedagogic College, Thai Nguyen University, Vietnam*

Received 20 March 2009; Accepted (in revised version) 09 June 2009

Available online 30 July 2009

Abstract. The solution of boundary value problems (BVP) for fourth order differential equations by their reduction to BVP for second order equations, with the aim to use the available efficient algorithms for the latter ones, attracts attention from many researchers. In this paper, using the technique developed by the authors in recent works we construct iterative method for a problem with complicated mixed boundary conditions for biharmonic equation which is originated from nanofluidic physics. The convergence rate of the method is proved and some numerical experiments are performed for testing its dependence on a parameter appearing in boundary conditions and on the position of the point where a transmission of boundary conditions occurs.

AMS subject classifications: 65N99, 65Z05, 76M25

Key words: Iterative method; biharmonic equation; mixed boundary conditions.

1 Introduction

The solution of fourth order differential equations by their reduction to BVP for the second order equations, with the aim of using available efficient algorithms for the latter ones, attracts attention from many researchers. Namely, for the biharmonic equation with the Dirichlet boundary condition, there has been intensive investigation on the iterative method, which leads the problem to two problems for the Poisson equation at each iteration (see, e.g., [8,9,11]). In 1992, Abramov and Ulijanova [1] proposed an iterative method for the Dirichlet problem for the biharmonic type equation, but the convergence of the method is not proved. In our previous works [3, 4, 6, 7] with the help of boundary or mixed boundary-domain operators introduced appropriately,

*Corresponding author.
Email: dangqa@ioit.ac.vn (Dang Q. A), letungsondhtn@yahoo.com (Le T. S.)

we constructed iterative methods for biharmonic and biharmonic type equations associated with the Dirichlet, Neumann or simple type of mixed boundary conditions. These iterative methods are originated from our earlier works [2,5].

In this work we develop our technique for a problem with rather complicated mixed conditions for biharmonic equation, namely, we consider the following problem

$$\Delta^2 u = f, \qquad \text{in } \Omega, \tag{1.1a}$$

$$\frac{\partial u}{\partial x} = g_1, \quad \frac{\partial \Delta u}{\partial x} = g_2, \qquad \text{on } \Gamma_1, \tag{1.1b}$$

$$u = g_3, \quad \frac{\partial u}{\partial y} + b\Delta u = g_4, \qquad \text{on } \Gamma_2, \tag{1.1c}$$

$$\frac{\partial u}{\partial x} = g_5, \quad \frac{\partial \Delta u}{\partial x} = g_6, \qquad \text{on } \Gamma_3, \tag{1.1d}$$

$$u = g_7, \qquad \text{on } \Gamma_4 \cup \Gamma_5, \tag{1.1e}$$

$$\frac{\partial u}{\partial y} - b\Delta u = g_8, \qquad \text{on } \Gamma_4, \tag{1.1f}$$

$$\Delta u = g_9, \qquad \text{on } \Gamma_5, \tag{1.1g}$$

where Ω is the rectangle $(0, l_1) \times (0, l_2)$, and $\Gamma_1, \cdots, \Gamma_5$ are parts of the boundary $\Gamma = \partial\Omega$ as shown in Fig. 1, Δ is the Laplace operator, f and g_i (i=1, $\cdots$, 9) are functions given in Ω and on parts of the boundary Γ, respectively, b =const$\geq$ 0.

This problem with special right hand sides in equation and boundary conditions describes the slip behaviour in liquid films on surfaces of partterned wettability (see [12]). For the problem in general setting (1.1), we propose an iterative method which reduces it to a sequence of problems for the Poisson equation. The convergence of the method will be established and the numerical experiments will confirm the efficiency of the method under investigation.

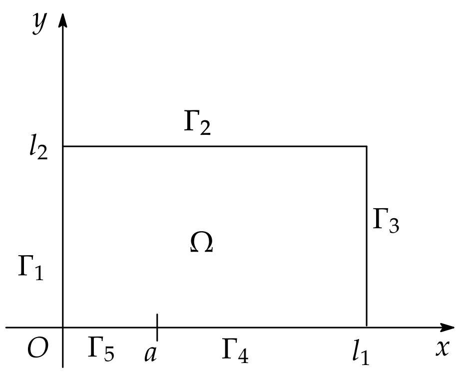

Figure 1: Domain Ω and parts of its boundary.