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# Investigation of Taylor-Görtler-like Vortices Using the Parallel Consistent Splitting Scheme

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**Abstract.** Symmetric Taylor-Görtler-like vortices at $Re$=3200 and 5000 in 3D rectangular cavities with a moving top lid are studied numerically and tested with a spanwise aspect ratio of $1:1:L$, where $L$=1, 2, 3. Solutions are obtained by solving the momentum equations and the continuity equations using the consistent splitting scheme. The code presented here was ported to the Parallel Interoperable Computational Mechanics System Simulator (PICMSS). Stable solutions are obtained as limit cases of the transients.

**AMS subject classifications**: 65M10, 78A48

**Key words**: Mixed finite element, consistent splitting scheme, PICMSS.

## 1 Introduction

Numerical modelling of the three-dimensional (3D) flow of an incompressible viscous fluid in a 3D rectangular cavity with a moving wall is the most recognized test problem for verifying the accuracy and effectiveness of a numerical algorithm. Many researchers gave a comparative analysis of the solutions of the test problem in the GAMM workshop [6] in 1991 and reported that for moderate Reynolds number, say $Re \geq 3200$ for the spanwise aspect ratio (SAR) 1:1:3 the flow is essentially complex and unsteady. One of the most striking features of the flow pattern, known as the Taylor-Görtler-like (TGL) vortices, that were probably first found experimentally in [10], occurred in the transitive direction. Since then, many algorithms have been used for

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finding supportive evidence for the appearance of TGL vortices, e.g. finite volume [3], finite difference [2], and lattice Boltzmann method [14].

In the present work, we report the results of a numerical integration with the mixed finite element (FE) formulation of the consistent splitting scheme (CSS), as proposed earlier by Shen and Guermond [8]. Based on our observations, with finer grid refinement, several pairs of symmetric TGL vortices are obtained and even better results are yielded. Parallel computing is an indispensable tool for producing long-time simulations of a large scale problem while increasing the degrees of freedom. The code developed here was ported to the Parallel Interoperable Computational Mechanics System Simulator (PICMSS) [15]. Our calculations have been performed by two choices of Reynolds numbers, *Re*=3200 and 5000, and three SARs of 1:1:1, 1:1:2 and 1:1:3. As we shall see below, not only are stable solutions obtained as limit cases of the transients, but also the behaviour of the pressure solution is free of node-to-node oscillations in this work.

The paper is divided into five sections; the first being the Introduction. Section 2 includes the general formulation of the problem and the physical problem with the boundary condition is described. Section 3 addresses the detailed numerical procedure for solving the 3D Navier-Stokes equations in the primitive variable form and the parallel computing engine, namely the PICMSS. Section 4 presents the results using the parallel mixed FE of the CSS solver and compares them with the other existing results. Section 5 gives the concluding remarks.

## 2 Formulation of the problem

The motion induced in a viscous incompressible fluid, contained in a 3D cavity of width $W$, length $L$ and height $H$ due to instantaneous motion of a sliding wall at a constant velocity is governed by the relation:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u}, \qquad \text{in } \Omega\times(0,T], \tag{2.1}$$

$$\nabla\cdot\boldsymbol{u} = 0, \qquad \text{in } \Omega\times[0,T], \tag{2.2}$$

where $\boldsymbol{u}=\boldsymbol{u}(\boldsymbol{x},t)=(u,v,w)$ is the non dimensional velocity, $p=p(\boldsymbol{x},t)$ is the non-dimensional pressure, $Re$ is the Reynolds number, $t$ is the time, and $\boldsymbol{x}=(x,y,z)$ is the spatial coordinate. A fixed final time is $T$. The bounded domain is

$$\Omega = [0,W]\times[0,L]\times[0,H].$$

Given an initial velocity field, which satisfies Eq. (2.2) and appropriate boundary conditions for $\boldsymbol{u}$, Eqs. (2.1) and (2.2) can be solved, in principal, for $\boldsymbol{u}$ and $p$ as functions of space and time.

It is well-known that several pairs of Taylor-Görtler-like vortices, experimentally and numerically speaking, appear on the lower wall when the Reynolds number increases and the SAR of the cavity (Width:Height:Length = $W : H : L$) varies. The