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Fourth Order Compact Boundary Value Method for Option Pricing with Jumps

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Abstract. We consider pricing options in a jump-diffusion model which requires solving a partial integro-differential equation. Discretizing the spatial direction with a fourth order compact scheme leads to a linear system of ordinary differential equations. For the temporal direction, we utilize the favorable boundary value methods owing to their advantageous stability properties. In addition, the resulting large sparse system can be solved rapidly by the GMRES method with a circulant Strang-type preconditioner. Numerical results demonstrate the high order accuracy of our scheme and the efficiency of the preconditioned GMRES method.

AMS subject classifications: 65T50, 65M06, 65F10, 91B28
Key words: Partial integro-differential equation, fourth order compact scheme, boundary value method, preconditioning, Toeplitz matrix.

1 Introduction

One of the most influential financial models is the jump-diffusion model presented by Merton [19] in 1976. In Merton's model, the asset return follows a standard Brownian process impelled by a compound Poisson process with normally distributed jumps. Under this assumption, the value of a contingent claim satisfies a partial integro-differential equation (PIDE). A PIDE usually comprises a differential operator and a non-local integral term. Numerical methods for solving PIDEs have already been widely studied [1, 2, 10–12, 15, 24]. However, the commonly used central difference discretization is only second order accurate in the spatial direction. Recently the authors of this paper proposed to apply a fourth order compact (FOC) finite difference scheme with local mesh refinement strategy to attain fourth order convergence in the spatial direction [18]. The approach in [18] is to firstly discretize the temporal direction of the PIDE by an implicit-explicit (IMEX) scheme. Then the semi-discretized equation

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at each time step can be naturally approximated by the FOC scheme. Though convenient to use, the IMEX scheme is only first order accurate. Therefore one has to employ the extrapolation strategy to reach high order accuracy for the time direction [12], and thus the operation cost depends highly on the number of extrapolation stages.

The unwanted workload of the above treatment pushes us to seek an alternative path. In 2003, Sun and Zhang [23] combined the boundary value method (BVM) with FOC scheme to solve one-dimensional heat equations. The main concept in [23] is as follows. After discretizing the derivatives of the spatial variable by FOC scheme, they utilize a high order BVM to approximate the semi-discretized linear system of ordinary differential equations (ODEs). The BVMs are a class of numerical methods based on the linear multistep formula (LMF) for solving initial value problems (IVPs) of ODEs [5,6]. In particular, the unconditional stability properties of BVMs make them preferable over other initial value methods (IVMs) [6]. Furthermore, one can obtain a high order BVM by implementing the LMF properly [5], i.e., there is no need to use any extrapolation strategy.

In this paper, we consider solving a PIDE by extending Sun-Zhang's idea. Unlike [18], we first carry out a three point FOC discretization for the spatial direction, and then a high order BVM is employed for the semi-discretized ODE system. This combination, known as the FOCBVM, retains high order accuracy and remarkable stability property at the same time. Unfortunately, we remark that the initial condition is always non-smooth in option pricing theory. In the spatial direction, a specific local mesh refinement strategy [18, 25] is required to ease the impact of the non-smooth payoff function and restore fourth order convergence of the FOC scheme. In the temporal direction, as stated in [13, 20, 24], a numerical correction process should be executed beforehand or numerical oscillations are likely to spoil the desired convergence. For a high order BVM, we can retrieve the expected result by replacing the approximations of the beginning time steps with the second order backward difference formula (BDF2). The BDF2 has been previously used by Almendral and Oosterlee [1] to achieve second order convergence in time, and they showed that the linear system at each time step can be swiftly solved by an iterative method based on the regular splitting of matrices.

Nevertheless, a direct solver for FOCBVM may not be a wise move because of the incredibly huge size of the resulting system. In fact, this is a major challenge for solving systems of LMF-based ODE codes. In 2000, Bertaccini [3] proposed to use the Krylov subspace method with block-circulant preconditioners to solve such systems. From then on, many circulant preconditioners are designed to pair with the GMRES method for the same purpose [8,16,17]. In this paper, we follow [8] 's idea to speed up calculation by constructing a Strang-type circulant preconditioner. We will see from the numerical results that the preconditioned GMRES method works very well.

The rest of the paper is organized as follows. In Section 2 we apply the FOC scheme with local mesh refinement strategy to discretize the spatial direction of the PIDE. In Section 3 we introduce the BVM implementation and the BDF2 startup procedure. The preconditioning technique is discussed in Section 4. In Section 5 we illustrate the