Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 1, pp. 104-116

DOI: 10.4208/aamm.2014.m642
February 2016

A High-Order Discontinuous Galerkin Method for the Two-Dimensional Time-Domain Maxwell's Equations on Curved Mesh

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Received 2 June 2014; Accepted (in revised version) 10 December 2014

Abstract. In this paper, a DG (Discontinuous Galerkin) method which has been widely employed in CFD (Computational Fluid Dynamics) is used to solve the two-dimensional time-domain Maxwell's equations for complex geometries on unstructured mesh. The element interfaces on solid boundary are treated in both curved way and straight way. Numerical tests are performed for both benchmark problems and complex cases with varying orders on a series of grids, where the high-order convergence in accuracy can be observed. Both the curved and the straight solid boundary implementation can give accurate RCS (Radar Cross-Section) results with sufficiently small mesh size, but the curved solid boundary implementation can significantly improve the accuracy when using relatively large mesh size. More importantly, this CFD-based high-order DG method for the Maxwell's equations is very suitable for complex geometries.

AMS subject classifications: 65Z05

Key words: Maxwell's equations, discontinuous Galerkin method, curved mesh, radar cross-section.

1 Introduction

In the last few decades, the finite difference time domain (FDTD) method has been widely used to solve the Maxwell's Equations in the time domain [1, 2], where the stair-stepped approximation of the curved boundary is usually employed which can affect the accuracy [3], particularly for complex geometries.

Finite element methods [4] have also been tried for solving the Maxwell's equations. Some bottlenecks such as the high cost due to large matrix inversion and the continuity

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requirement over element interfaces limited its applications in high-order cases. The Finite Volume (FV) methods [5, 6], which have been widely used in Computational Fluid Dynamics (CFD), are also suitable for solving the Maxwell's equations. However, the main disadvantage of the FV methods is their low order property, which makes it necessary to increase the number of grid elements to ensure the numerical accuracy.

Over the past decade, Discontinuous Galerkin methods [7] have received growing interests in solving time-domain Maxwell's equations because of their advantages in implementing upwinding, hp-adaptivity and parallelization. In [8], DG was used to solve the dispersive lossy Maxwell's equations in PML (Perfectly Matched Layer) regions. A spatial high-order hexahedral DG was introduced in [9], where comparisons against the FDTD and the FVTD are given. The efficiency improvement of DG by using hybrid meshes is displayed in [10]. [11] discussed Petrov-Galerkin and DG methods for both time-domain and frequency-domain electromagnetic calculations. [12] introduced a non-conforming multi-element DG for irregular geometries, where unstructured triangulation is used near objects and structured quadrangulation for the rest. [13] developed two hybridizable DG methods for time-harmonic Maxwell's equations. The convergence and superconvergence of staggered DG for Maxwell's equations on Cartesian mesh were analyzed in [14]. [15, 16] and [17] developed DG methods for Maxwell's equations in meta-materials and anisotropic materials. [18] discussed a Schwarz-type domain decomposition method when solving the 3D Maxwell's equations with DG. A *hp*-adaptivity DG was employed to perform large scale electromagnetic simulations in [19].

In this paper, we aim to apply the CFD-based high-order DG to solve the Maxwell's equations in very complex geometry cases. The paper is organized as follows. In Section 2, the CFD-based high-order DG discretization is described, where the quadrature-free implementation and parallel computing are employed to save the CPU time. In Section 3, a high-order approximation of solid boundary is introduced to approach the real geometries. Numerical results are displayed in Section 4 and the paper ends with the conclusions in Section 5.

2 High-order DG discretization of the time-domain Maxwell's equations

In the case of Transverse Magnetic (TM) wave, the 2D Maxwell's equations in the conservation form can be written as:

$$\frac{\partial \mathbf{U}}{\partial t}+\nabla\cdot\mathbf{F}(\mathbf{U})=\mathbf{S}(\mathbf{U}), \tag{2.1}$$

where

$$\mathbf{U}=\begin{bmatrix}\epsilon E_z\\ \mu H_x\\ \mu H_y\end{bmatrix},\quad \mathbf{F}(\mathbf{U})=(\mathbf{F}^x,\mathbf{F}^y)\quad\text{and}\quad \mathbf{F}^x=\begin{bmatrix}-H_y\\ 0\\ -E_z\end{bmatrix},\quad \mathbf{F}^y=\begin{bmatrix}H_x\\ E_z\\ 0\end{bmatrix}.$$