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Reduction of Numerical Oscillations in Simulating Moving-Boundary Problems by the Local DFD Method

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Abstract. In this work, the hybrid solution reconstruction formulation proposed by Luo et al. [H. Luo, H. Dai, P. F. de Sousa and B. Yin, On the numerical oscillation of the direct-forcing immersed-boundary method for moving boundaries, Computers & Fluids, 56 (2012), pp. 61–76] for the finite-difference discretization on Cartesian meshes is implemented in the finite-element framework of the local domain-free discretization (DFD) method to reduce the numerical oscillations in the simulation of moving-boundary flows. The reconstruction formulation is applied at fluid nodes in the immediate vicinity of the immersed boundary, which combines weightly the local DFD solution with the specific values obtained via an approximation of quadratic polynomial in the normal direction to the wall. The quadratic approximation is associated with the no-slip boundary condition and the local simplified momentum equation. The weighted factor suitable for unstructured triangular and tetrahedral meshes is constructed, which is related to the local mesh intervals near the immersed boundary and the distances from exterior dependent nodes to the boundary. Therefore, the reconstructed solution can account for the smooth movement of the immersed boundary. Several numerical experiments have been conducted for two- and three-dimensional moving-boundary flows. It is shown that the hybrid reconstruction approach can work well in the finite-element context and effectively reduce the numerical oscillations with little additional computational cost, and the spatial accuracy of the original local DFD method can also be preserved.

AMS subject classifications: 76D05, 76M99

Key words: Numerical oscillation, immersed boundary method, moving boundary, domain-free discretization, boundary condition.

1 Introduction

Recently, the immersed boundary methods (IBMs) have gained a special attention for its simplicity and effectiveness in the simulation of flows with complex moving boundaries.

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According to the classification proposed by Mittal and Iaccarino [1], IBMs can be categorized into two major groups. One is the so-called continuous forcing approach where the forcing is incorporated into the momentum equations before discretization is performed. The distinguished drawback of this approach is that the boundary condition on the immersed interface cannot be precisely satisfied and the interface is smeared. The other group corresponds to the discrete forcing approach where the boundary condition is directly introduced into the discrete equations by imposing discrete body forces, explicitly or implicitly, on the cells/nodes close to the surface of immersed body and thereby enables a sharp representation of the immersed interface.

Temporal oscillations can be found in the vast majority of the numerical results obtained by the discrete-forcing IBMs [2–5]. Lee et al. [3] indicated that the spatial discontinuity of pressure and the temporal discontinuity of velocity are the two sources of these numerical oscillations. In the work of Kim et al. [4], the oscillations are found to be reduced by introducing the mass source/sink terms. In the work of Seo and Mittal for the ghost-cell IBM [2], the numerical oscillations are reduced by adopting the Cartesian cut-cell approach [6,7] with a virtual merging technique [8]. Later, Lee and You [9] proposed a fully-implicit ghost-cell IBM coupled with a mass source/sink term to remedy this problem. Nevertheless, all the aforementioned approaches are considered from the perspective of improving mass conservation near the immersed boundary. To achieve this goal, a tedious task of geometric handling, such as local reconstruction of mesh cells in the immediate vicinity of the boundary and recalculation of fluxes for these cells is inevitably required. These processes could be time-consuming and difficult to implement, especially for three-dimensional problems with complex geometries. Recently, Luo et al. [10] proposed a hybrid reconstruction formulation which is considered from the perspective of making smooth transitions of numerical descriptions at the direct-forcing points. They stated that when the boundary moves across the nodes on the fixed background mesh, the abrupt change of numerical description between the standard finite-difference formula and the flow reconstruction at these nodes could cause numerical oscillations in the pressure. To reduce the oscillations, a hybrid formulation was proposed, which combines the reconstructed solution and the solution of governing equations at the fluid nodes immediately next to the solid boundary.

Zhou and Shu recently proposed a local domain-free discretization (DFD) method to simulate the moving-boundary flows [11]. In this method, a partial differential equation is discretized at all mesh nodes inside the solution domain, but the discrete form may involve some exterior nodes. The exterior dependent nodes, which are connected to an interior node by a cell edge, serve as the role to enforce the wall boundary condition. The flow variables at an exterior dependent node are obtained via some approximate form of the solution in the vicinity of solid boundary. This method can be classified as a discrete-forcing IBM. Unsurprisingly, temporal oscillations can be observed in the previous DFD results of moving-boundary problems.

Following the work of Luo et al. [10], we formulate a hybrid solution reconstruction in the finite-element framework of the local DFD method to reduce the numerical