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Improving the Stability of the Multiple-Relaxation-Time Lattice Boltzmann Method by a Viscosity Counteracting Approach

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Abstract. Numerical instability may occur when simulating high Reynolds number flows by the lattice Boltzmann method (LBM). The multiple-relaxation-time (MRT) model of the LBM can improve the accuracy and stability, but is still subject to numerical instability when simulating flows with large single-grid Reynolds number (Reynolds number/grid number). The viscosity counteracting approach proposed recently is a method of enhancing the stability of the LBM. However, its effectiveness was only verified in the single-relaxation-time model of the LBM (SRT-LBM). This paper aims to propose the viscosity counteracting approach for the multiple-relaxation-time model (MRT-LBM) and analyze its numerical characteristics. The verification is conducted by simulating some benchmark cases: the two-dimensional (2D) lid-driven cavity flow, Poiseuille flow, Taylor-Green vortex flow and Couette flow, and three-dimensional (3D) rectangular jet. Qualitative and Quantitative comparisons show that the viscosity counteracting approach for the MRT-LBM has better accuracy and stability than that for the SRT-LBM.

AMS subject classifications: 65M10, 78A48

Key words: Multiple-relaxation-time lattice Boltzmann method, viscosity counteracting, high Reynolds number flow, Poiseuille flow, Couette flow, Taylor-Green vortex flow, lid-driven cavity flow.

1 Introduction

The lattice Boltzmann method (LBM) has some unique advantages for simulating various complex flow problems [1]. Among many models of the LBM, the single-relaxation-time (SRT) model (with the Bhatnagar-Gross-Krook collision operator) has been popular because it has a simpler collision term [2]. However, this model becomes unstable when

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simulating high Reynolds-number flows [3]. Many efforts have been made to improve the stability of the LBM. Hou improved the robustness of the SRT model by introducing a large eddy model for turbulence simulations [4]. He proposed a multi-grid method that imposes curvilinear coordinates near the boundary walls to increase the Reynolds number in simulated problems [5]. Zhang improved the numerical stability by introducing fractional volume in the LBM [6]. Shu suggested a step-by-step LBM that combines the standard LBM with the fractional step method [7]. A significant improvement of the accuracy and stability of LBM was not achieved until the establishment of the multiple-relaxation-time (MRT) model [8], which was theoretically analyzed in detail by Lallemand and Luo in 2000 [9]. In the MRT model, the collision process is conducted in the moment spaces, and different relaxation times correspond to different physical variables. The numerical stability and accuracy of MRT model is much better than other lattice Boltzmann models [10,14,21], and the model provides much richer physical information of the fluid field. According to some reports, the simulation costs are 15% higher than the SRT model, but the stability and accuracy benefits override the computational expense [11]. The MRT model has been widely used to simulate flows, especially flows of high Reynolds number [12–14]. However, the MRT model becomes unstable when the single-grid Reynolds number is very high if no additional effort, such as the subgrid-scale modeling, is adopted [15].

Recently, a viscosity counteracting approach (VC) was proposed by Cheng et al. for improving stability of the SRT-model in simulations of high Reynolds number flows [16]. The main idea is to counteract the artificial viscosity corresponding to a flow of greater viscosity introduced intentionally to simulate an actual flow with smaller viscosity in a stable way by adding an external foring term to the equations. The enhanced stability and accuracy of this approach were verified on typical benchmarks. There are still rooms for improving its effectiveness. Because the approach works well for SRT-LBM and MRT model has better stability and accuracy than SRT model, one may anticipate that better performance can be achieved by introducing the viscosity counteracting approach into MRT model.

The aim of this study is to develop a viscosity counteracting approach for the MRT-LBM and verify its effectiveness. Section 2 describes the basic theory of the MRT model and the principles of the proposed approach. Section 3 addresses the verification of the approach by some benchmarks: 2D Poiseuille flow, Couette flow, Taylor-Green vortex flow and lid-driven cavity flow, and 3D rectangular jet. Section 4 concludes the paper.

2 Methods

2.1 Multiple-relaxation-time model

Without loss of generality, we consider the two-dimensional-nine-velocity (D2Q9) model