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Nonlinear Vibration Analysis of Functionally Graded Nanobeam Using Homotopy Perturbation Method

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Abstract. In this paper, He's homotopy perturbation method is utilized to obtain the analytical solution for the nonlinear natural frequency of functionally graded nanobeam. The functionally graded nanobeam is modeled using the Eringen's nonlocal elasticity theory based on Euler-Bernoulli beam theory with von Karman nonlinearity relation. The boundary conditions of problem are considered with both sides simply supported and simply supported-clamped. The Galerkin's method is utilized to decrease the nonlinear partial differential equation to a nonlinear second-order ordinary differential equation. Based on numerical results, homotopy perturbation method convergence is illustrated. According to obtained results, it is seen that the second term of the homotopy perturbation method gives extremely precise solution.

AMS subject classifications: 74G10, 74H45, 74E30,74B99

Key words: Homotopy perturbation method, Lindstedt-Poincare method, analytical solution, nonlocal nonlinear free vibration, functionally graded nanobeam.

1 Introduction

Functionally graded (FG) materials are a new class of composite materials. These composite include of two or more materials, in which the material properties change smoothly from one interface to the other. During the past years, FG materials have a great practical importance because of their wide applications in many industrial and engineering fields. Recently, the application of FG materials has widely been developed in nano-structures such as nano-electromechanical systems, spacecraft heat shields, thin films in the form of shape memory alloys, plasma coatings for fusion reactors, and atomic force microscopes, jet fighter structures to obtain high sensitivity and desired function.

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Due to the presence of small-scale effects at the nanoscale, the classical continuum theories fail to accurately predict the mechanical behavior of nanostructures [1]. So, nonlocal continuum theories which contain additional material length scale parameter have been proposed to predict the accurate behavior of nano-structures. One widely promising size-dependent continuum theory is the nonlocal elasticity theory pioneered by Eringen [2].

The approximate analytical methods have their own restrictions. For example, Perturbation method depends on small parameter and choosing unsuitable small parameter can be lead to wrong solution [3]. Homotopy is an important part of topology [4] and it can convert any nonlinear problem to a finite linear problems and it doesn't depend on small parameter [5,6]. Lia [7] based on homotopy proposed homotopy analysis method (HAM). The homotopy perturbation method was first proposed by Ji-Huan He [8] in 1999 for solving differential and integral equations. Homotopy perturbation method (HPM) is a combination of homotopy and classic perturbation techniques. The HPM has a significant advantage that it provides an analytical approximate solution to a wide range of nonlinear problems in applied sciences. HPM is a special case of HAM that due to its easier algorithm, it is used in this paper.

In practical, the governing differential equations of many vibrational systems are nonlinear such as satellites, helicopter blades, airplane wings, towers. They have no exact solution generally. Consequently, numerical or approximate analytical methods are used to investigate behavior of nonlinear systems. Numerical method like boundary element and finite element methods don't give parametric solutions. Hence, they have no application to study the qualitative and global response of the vibrational systems. Some approaches such as perturbation methods can eliminate shortages of numerical methods. There are some approaches to solve the governing equations of the nonlinear vibrations such as homotopy and perturbation methods and combination of them. We cite some of the papers that were used these methods.

Foda [9] and Ramezani et al. [10] used multiple scales to investigate the nonlinear vibration of the beam by considering shear deformation and rotary inertia effects for both sides simply supported (SS) and clamped-clamped, respectively. Nazemnezhad et al. [11] utilized the same method to investigate the nonlinear natural frequency of the FG nanobeam with considering small scale. The perturbation method was used to study the nonlinear vibrations of beams with different boundary conditions by Evensen [12]. Pirbodaghi et al. [13] studied nonlinear vibrational behavior of Euler Bernoulli beams subjected to axial loads using HAM. Akbarzade et al. [14] presented a new technique couples HPM with variational method for solving approximate analytical higher order solutions for strong nonlinear Duffing oscillators with cubic-quintic nonlinear restoring force. Bayat et al. [15] analyzed the high amplitude free vibrations of the tapered beams by using Max-Min Approach (MMA) and HPM. Ahmadian et al. [16] utilized homotopy and modified Lindstedt-Poincare methods to study the nonlinear free vibrations of the beams subjected to axial loads. Moeenfard et al. [17] used the same method to study the nonlinear natural frequencies of the pre-stretched microbeam considering the effects of rotary inertia and shear deformation. Poorjamshidian et al. [18] employed combination of homotopy and traditional perturbation methods in investigation of the nonlinear vibration in large amplitude for SS beam with a constant velocity carrying a moving mass. Yazdi [19] studied the nonlinear natural frequency of doubly curved cross-ply shells with SS boundary conditions by using HPM. Using HPM, Sedighi et al. [20] studied transversal oscillation of quintic nonlinear beam. Sedighi et al. [21] presented homotopy perturbation method with an auxiliary term (HPMAT) and they used HPMAT to study buckled beam nonlinear vibration, uniform beam carrying an intermediate lumped mass and transversely vibrating quintic nonlinear beam.

In this work, free vibration of FG nanobeam is investigated in the context of the nonlocal continuum theory of Eringen and by employing Von Karman type nonlinearity within frame work of Euler-Bernoulli beam theory. An approximation based on the Galerkin method is used to reduce the partial differential equation (PDE) of motion and associated boundary conditions to a system of nonlinear second-order ordinary differential equation (ODE). The HPM is used to obtain approximate analytical solutions for nonlinear natural frequency.

2 Governing equation

Utilizing the nonlocal elasticity within the frame work of Euler-Bernoulli beam theory (EBT) with von Karman type nonlinearity the nonlocal nonlinear free vibration is governed by the following nonlinear PDE [11]

$$(P)\frac{\partial^4 \hat{W}}{\partial \hat{x}^4}+(K)\frac{\partial^2 \hat{W}}{\partial \hat{x}^2}-I_1\frac{\partial^2 \hat{W}}{\partial t^2}+\mu I_1\frac{\partial^4 \hat{W}}{\partial \hat{x}^2\partial \hat{t}^2}=0. \tag{2.1}$$

Where

$$P=\frac{bB_1^2}{A_1}-bC_1-\mu\frac{bA_1}{2L}\int_0^L\Big(\frac{\partial \hat{W}}{\partial \hat{x}}\Big)^2dx+\mu\frac{bB_1}{L}\int_0^L\Big(\frac{\partial^2 \hat{W}}{\partial \hat{x}^2}\Big)dx, \tag{2.2a}$$

$$K=\frac{bA_1}{2L}\int_0^L\Big(\frac{\partial \hat{W}}{\partial \hat{x}}\Big)^2dx-\frac{bB_1}{L}\int_0^L\Big(\frac{\partial^2 \hat{W}}{\partial \hat{x}^2}\Big)dx, \tag{2.2b}$$

$$\{A_1,B_1,C_1\}=\int_{-\frac{h}{2}}^{+\frac{h}{2}}E(z)\{1,z,z^2\}dz, \qquad I_1=\int_{-\frac{h}{2}}^{+\frac{h}{2}}b\rho(z)dz. \tag{2.2c}$$

It is assumed that material properties of the nanobeam, such as Young's modulus $E(z)$ and mass density $\rho(z)$, vary continuously through the beam thickness according to power law distribution form, which can be described by

$$E(z)=(E_1-E_2)\Big(\frac{2z+h}{2h}\Big)^m+E_2, \tag{2.3a}$$

$$\rho(z)=(\rho_1-\rho_2)\Big(\frac{2z+h}{2h}\Big)^m+\rho_2. \tag{2.3b}$$

Where the subscripts 1 and 2 denote the top surface and the bottom surface, respectively, and a gradient index m determines the variation profile of material properties across the FG nanobeam thickness. In this equation, L is the length, h is thickness, b is width of the FG nanobeam and μ is the small-scale parameter includes influence of length scale.

3 Problem solution

Upon employing the dimensionless quantities as follows:

$$t=\frac{\hat{t}}{T_n}, \tag{3.1a}$$

$$x=\frac{\hat{x}}{L}, \tag{3.1b}$$

$$W=\frac{\hat{W}}{L}. \tag{3.1c}$$

Where

$$T_n=\sqrt{\frac{I_1L^4}{\left(bC_1-\frac{bB_1^2}{A_1}\right)}}. \tag{3.2}$$

The governing equation of motion of the FG beam can be expressed as the following normalized form:

$$\begin{aligned}&\frac{\partial^4 W}{\partial x^4}+\frac{I_1L^4}{T_n^2\left(bC_1-\frac{bB_1^2}{A_1}\right)}\frac{\partial^2 W}{\partial t^2}+\frac{\mu bA_1}{2\left(bC_1-\frac{bB_1^2}{A_1}\right)}\frac{\partial^4 W}{\partial x^4}\int_0^1\left(\frac{\partial W}{\partial x}\right)^2dx\\&\quad-\frac{\mu bB_1}{L\left(bC_1-\frac{bB_1^2}{A_1}\right)}\frac{\partial^4 W}{\partial x^4}\int_0^1\left(\frac{\partial^2 W}{\partial x^2}\right)dx-\frac{bA_1L^2}{2\left(bC_1-\frac{bB_1^2}{A_1}\right)}\frac{\partial^2 W}{\partial x^2}\int_0^1\left(\frac{\partial W}{\partial x}\right)^2dx\\&\quad+\frac{bB_1L}{\left(bC_1-\frac{bB_1^2}{A_1}\right)}\frac{\partial^2 W}{\partial x^2}\int_0^1\left(\frac{\partial^2 W}{\partial x^2}\right)dx-\frac{\mu I_1L^2}{\left(bC_1-\frac{bB_1^2}{A_1}\right)T_n^2}\frac{\partial^4 W}{\partial x^2\partial t^2}=0.\end{aligned} \tag{3.3}$$

Considering the first mode shape of linear vibration, it is generally the dominant mode shape of the vibration, the solution of Eq. (3.3) can be assumed as:

$$W(x,t)=\phi(x)\cdot q(t). \tag{3.4}$$

Where $\phi(x)$ is the normalized form of the first linear mode shape and can be stated as

$$\phi(x)=\frac{\mathcal{Q}(x)}{\max\{\mathcal{Q}(x)\}}. \tag{3.5}$$

Where $Q(x)$ is defined as follows:

$$SS:\ \ \mathcal{Q}(x)=\sin(n\pi x). \tag{3.6}$$

Simply supported-clamped (SC):

$$\mathcal{Q}(x)=\begin{cases} \sin(\zeta_1 x)-\dfrac{\sin(\zeta_1)}{\sinh(\zeta_2)}\sinh(\zeta_2 x),\\ \zeta_{1,2}=\Big(\dfrac{\pm b+\sqrt{b^2+4a}}{2}\Big)^{0.5},\\ H=bC_1-\dfrac{bB_1^2}{A_1},\quad a=\dfrac{I_1\omega^2}{H},\quad b=\dfrac{I_1\mu\omega^2}{H}. \end{cases} \tag{3.7}$$

The parameters ζ_1 and ζ_2 can be determined from the eigenvalue equation

$$\zeta_2\tan(\zeta_1)-\zeta_1\tanh(\zeta_2)=0.$$

Note that $\phi(x)$ satisfies the geometric and forcing boundary conditions of the beam, which can be described as

$$SS:\ \begin{cases} W(0,t)=0,\\ \dfrac{\partial^2 W}{\partial x^2}(0,t)=0,\\ W(1,t)=0,\\ \dfrac{\partial^2 W}{\partial x^2}(1,t)=0, \end{cases} \tag{3.8a}$$

$$SC:\ \begin{cases} W(0,t)=0,\\ \dfrac{\partial^2 W}{\partial x^2}(0,t)=0,\\ W(1,t)=0,\\ \dfrac{\partial W}{\partial x}(1,t)=0. \end{cases} \tag{3.8b}$$

The initial conditions are assumed as:

$$W(x,0)=\frac{q_{\max}}{r}\phi(x), \tag{3.9a}$$

$$\frac{\partial W}{\partial t}(x,0)=0. \tag{3.9b}$$

Where r and $q_{\max}$ are gyration radius and vibration amplitude, respectively.

Substituting Eq. (3.4) in to Eq. (3.3) and using Galerkin's procedure and integrating the residual by weight $\phi(x)$ over the problem domain, the nonlinear ordinary differential for the first vibrational mode can be derived as

$$\ddot{q}+\alpha_1 q+\alpha_2 q^2+\alpha_3 q^3=0. \tag{3.10}$$

Where

$$\alpha_1=\frac{F_3}{\left(F_1-\frac{\mu}{L^2}F_2\right)}, \tag{3.11a}$$

$$\alpha_2=\frac{bB_1LF_2F_4-\frac{\mu bB_1}{L}F_3F_4}{\left(bC_1-\frac{bB_1^2}{A_1}\right)\left(F_1-\frac{\mu}{L^2}F_2\right)}, \tag{3.11b}$$

$$\alpha_3=\frac{\mu bA_1F_3F_5-bA_1L^2F_2F_5}{2\left(bC_1-\frac{bB_1^2}{A_1}\right)\left(F_1-\frac{\mu}{L^2}F_2\right)}. \tag{3.11c}$$

In Eqs. (3.11a) to (3.11c), F_i, $1\le i\le 5$ are described as follows:

$$F_1=\int_0^1\phi^2dx, \tag{3.12a}$$

$$F_2=\int_0^1\phi''\phi dx, \tag{3.12b}$$

$$F_3=\int_0^1\phi''''\phi dx, \tag{3.12c}$$

$$F_4=\int_0^1\phi'' dx, \tag{3.12d}$$

$$F_5=\int_0^1(\phi')^2dx. \tag{3.12e}$$

Homotopy perturbation is used to solve Eq. (3.10). The initial conditions (3.9a) and (3.9b) can be translated to the following initial condition for $q(t)$

$$q(0)=\frac{q_{\max}}{r}, \tag{3.13a}$$

$$\dot{q}(0)=0. \tag{3.13b}$$

The homotopy form is constructed as follows:

$$(1-P)(\ddot{q}+\alpha_1q)+P(\ddot{q}+\alpha_1q+\alpha_2q^2+\alpha_3q^3)=0, \tag{3.14}$$

Eq. (3.14) can be simplified as

$$\ddot{q}+\alpha_1q+P(\alpha_2q^2+\alpha_3q^3)=0. \tag{3.15}$$

From Eq. (3.15), linear frequency of the FG beam are calculated as

$$\omega_{10}^2=\alpha_1. \tag{3.16}$$

Using the modified Lindstedt-Poincare method [22,23], $q(t)$, $\omega_1 0^2$ are perturbed utilizing homotopy parameter P

$$q(t)=q_0(t)+Pq_1(t)+\mathcal{O}(P^2), \tag{3.17a}$$

$$\omega_{10}^2=\omega_1^2+P\omega_{11}+\mathcal{O}(P^2). \tag{3.17b}$$

Substituting Eqs. (3.17a) and (3.17b) into Eq. (3.15) and setting the coefficients of each power of P to zero, it yields the following set of equations:

$$\ddot{q}_0+\omega_1^2 q_0=0, \tag{3.18a}$$

$$\ddot{q}_1+\omega_1^2 q_1+\omega_{11} q_0+\alpha_2 q_0^2+\alpha_3 q_0^3=0. \tag{3.18b}$$

Initial conditions (3.13a) and (3.13b) are converted as

$$q_0(0)=\frac{q_{\max}}{r}, \tag{3.19a}$$

$$\dot{q}_0(0)=0, \tag{3.19b}$$

$$q_1(0)=0, \tag{3.19c}$$

$$\dot{q}_1(0)=0. \tag{3.19d}$$

Solving Eq. (3.18a) results

$$q_0(t)=A\cos\omega_1 t+B\sin\omega_1 t. \tag{3.20}$$

Where $A=q_{\max}/r$, $B=0$.

Substituting q_0 from Eq. (3.20) into Eq. (3.18b) and simplification leads to

$$\begin{aligned}&\ddot{q}_1+\omega_1^2 q_1+\left(\omega_{11}\frac{q_{\max}}{r}+\frac{3}{4}\alpha_3\left(\frac{q_{\max}}{r}\right)^3\right)\cos\omega_1 t\\ &\quad+\alpha_2\left(\frac{q_{\max}}{r}\right)^2\left(\frac{1+\cos 2\omega_1 t}{2}\right)+\alpha_3\left(\frac{q_{\max}}{r}\right)^3\frac{\cos 3\omega_1 t}{4}=0.\end{aligned} \tag{3.21}$$

Avoiding secular term in Eq. (3.21) gives

$$\omega_{11}\frac{q_{\max}}{r}+0.75\alpha_3\left(\frac{q_{\max}}{r}\right)^3=0. \tag{3.22}$$

Letting $P=1$ in Eq. (3.17b), we have

$$\alpha_1=\omega_1^2+\omega_{11}. \tag{3.23}$$

Substituting ω_{11} from Eq. (3.22) into Eq. (3.23), we have nonlocal nonlinear natural frequency of FG beam as

$$\omega_1^2=\alpha_1+0.75\alpha_3\left(\frac{q_{\max}}{r}\right)^2. \tag{3.24}$$

4 Results

In order to ensure of the present results, three comparison studies are conducted for this purpose. Consider a SS homogenous nanobeam in the presence of the nonlocal effect.

Comparison of the first linear nondomensional natural frequency between present and published work is illustrated in Table 1 for a SS nonlocal Euler-Bernoulli nanobeam with the parameters: $L=10$nm, $E=3010^6$Pa, $\rho=1$Kg/m^3. It is noted that obtained results have identical agreement with the analytical results given by [11, 24] for various small-scale parameters.

To illustrating convergence of HPM, nonlinear frequency to classical linear frequency ratios (ω_{nl}/ω_l) for the present HPM solution with various order (P^1,P^2,P^3), multiple scale method [11], exact solution [25] and the Ritz-Galerkin method [26] with $L=20$nm and $h=0.1$L, are given in Table 2. According to Table 2 speed of convergence of the HPM is very fast, and the accuracy of the method increases with increasing order of HPM solution and also a good agreement with the [11, 25, 26] and present results is observed. It is interesting to note that the results obtained by first order of present method exactly match with that of the solution obtained by Singh et al. using Ritz-Galerkin method [26] and second order of present result are match with exact solution [25].

In the next comparison study a FG nanobeam with squared cross-section ($b=h=0.1$L) composed of Aluminum (Al) and Silicon (Si) is considered as comparative study between the present HPM solution and [11] that the top surface $(z=h/2)$ and the bottom surface $(z=-h/2)$ of the FG nanobeam are silicon-rich and Aluminum-rich, respectively, and its mixture changes through the thickness.

The bulk elastic properties of aluminum with crystallographic direction of (111) and silicon with crystallographic direction of (100) are as follows:

$$\begin{cases} \text{Silicon [27]}: \ E_1=210GPa, \ \nu_1=0.24, \ \rho_1=2370\text{Kg/m}^3, \\ \text{Aluminum [28]}: \ E_2=70GPa, \ \nu_2=0.3, \ \rho_2=2700\text{Kg/m}^3. \end{cases}$$

To illustrate nonlocal effect on natural frequency we defined linear and nonlinear fre-

Table 1: Comparative study for the first linear nondimensional natural frequency of a SS isotropic nanobeam.

μ (nm^2)	Present	[11]	[24]
0	9.8696	9.8696	9.8696
1	9.4159	9.41588	9.4159
2	9.0195	9.01948	9.0195
3	8.6693	8.66927	8.6693
4	8.3569	8.35692	8.3569

Table 2: Convergence illustration and comparative study for the nonlinear frequency ratio (ω_{nl}/ω_l) of a SS isotropic nanobeam.

$q_{\max}/r$	P^1	P_2	P^3	[11]	[25]	[26]
1	1.0897	1.0892	1.0892	1.0937	1.0892	1.0897
2	1.3229	1.3178	1.3178	1.3750	1.3178	1.3229
3	1.6394	1.6256	1.6256	1.8438	1.6257	1.6394

Table 3: Frequency ratios of FG nanobeam for boundary condition SC and different amplitude ratios, gradient index, nonlocal parameter values, nanobeam length.

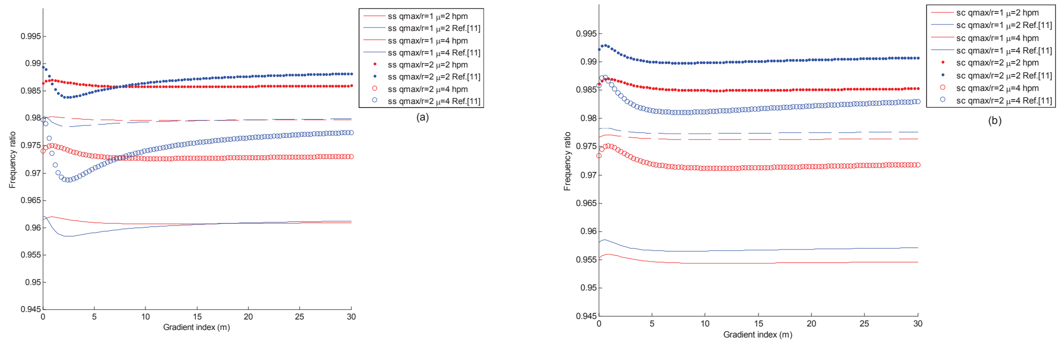
$q_{\max}/r$	m	L (nm)	[11] μ (nm^2) 2	[11] μ (nm^2) 4	HPM μ (nm^2) 2	HPM μ (nm^2) 4
0	3	10	0.9013	0.8267	0.9013	0.8267
		20	0.9724	0.9469	0.9724	0.9469
		30	0.9874	0.9753	0.9874	0.9753
1	0	10	0.9202	0.8625	0.9172	0.8556
		20	0.9774	0.9567	0.9767	9553
		30	0.9897	0.9798	0.9894	0.9792
	1	10	0.9205	0.8625	0.9185	0.8579
		20	0.9775	0.9569	0.9771	0.9560
		30	0.9898	0.9799	0.9896	0.9795
	2	10	0.9192	0.8598	0.9178	0.8566
		20	0.9772	0.9562	0.9769	0.9556
		30	0.9896	0.9796	0.9895	0.9793
	3	10	0.9209	0.8632	0.9170	0.8552
		20	0.9776	0.9571	0.9767	0.9552
		30	0.9898	0.9800	0.9894	0.9791
2	0	10	0.9676	0.9519	0.9512	0.9158
		20	0.9899	0.9812	0.9861	0.9735
		30	0.9953	0.9909	0.9937	0.9876
	1	10	0.9680	0.9509	0.9543	0.9212
		20	0.9901	0.9816	0.9870	0.9752
		30	0.9954	0.9912	0.9941	0.9884
	2	10	0.9639	0.9426	0.9526	0.9183
		20	0.9891	0.9796	0.9865	0.9742
		30	0.9950	0.9903	0.9939	0.9880
	3	10	0.9700	0.9594	0.9507	0.9149
		20	0.9907	0.9827	0.9860	0.9732
		30	0.9957	0.9916	0.9936	0.9875

quency ratio as

$$\text{Nonlinear frequency ratio} = \frac{\text{(Nonlocal nonlinear natural frequency)}}{\text{(Classical nonlinear natural frequency)}},$$
$$\text{Linear frequency ratio} = \frac{\text{(Nonlocal linear natural frequency)}}{\text{(Classical linear natural frequency)}}.$$

Table 3 and Table 4 contain a comparison between the present results with [11] for nonlinear frequency ratio and linear frequency ratio for various small scale values ($\mu = 2{,}4\text{nm}^2$) and different nanobeam length ($L = 10{,}20{,}30\text{nm}$) and different gradient index when boundary conditions are both SC and SS, respectively.

Figs. 1(a) and (b) compare variations of the first nonlinear frequency ratios on the



(a) (b)

Figure 1: Variations of the first nonlinear frequency ratios on the gradient index, various amplitude ratios ($q_{\max}/r=1,2$), and different small scale values ($\mu=2,4\text{nm}^2$) when the length of nanobeam is $L=20$nm, (a) SS boundary conditions (b) SC boundary conditions.

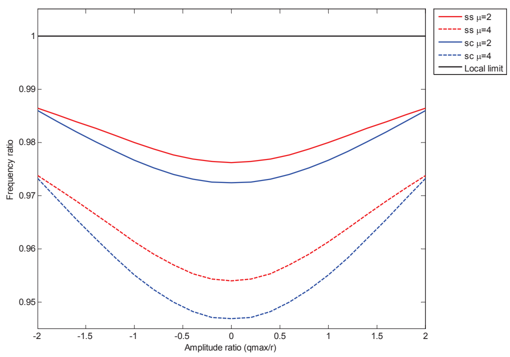


Figure 2: Variations of first frequency ratio on the amplitude ratio for different small scale values ($\mu=2,4\text{nm}^2$) with $L=20$nm and $m=3$.

gradient index between present study and [11] at selected amplitude ratios ($q_{\max}/r=1,2$), nonlocal parameters ($\mu=2,4\text{nm}^2$), FG nanobeam length $L=20$nm and when boundary conditions are both SS and SC respectively.

Fig. 2 displays variations of the first frequency ratios on the amplitude ratio for different values of small scale ($\mu=2,4\text{nm}^2$) and both SS and SC boundary conditions when the values of the FG nanobeam length $L=20$nm and the gradient index $m=3$. Some selected data from Fig. 2 are also listed in Table 5.

5 Conclusions

The aim of this paper is to investigate the application of homotopy perturbation method (HPM) in predicting nonlocal nonlinear natural frequency of a functionally graded (FG)

Table 4: Frequency ratios of FG nanobeam for boundary condition SS and different amplitude ratios, gradient index, nonlocal parameter values, nanobeam length.

$q_{\max}/r$	m	L (nm)	[11] μ (nm^2)		HPM μ (nm^2)	
			2	4	2	4
0	3	10	0.9139	0.8467	0.9139	0.8467
		20	0.9762	0.9540	0.9762	0.9540
		30	0.9892	0.9788	0.9892	0.9788
1	0	10	0.9293	0.8754	0.9280	0.8727
		20	0.9803	0.9621	0.9800	0.9614
		30	0.9911	0.9824	0.9909	0.9821
	1	10	0.9247	0.8659	0.9291	0.8747
		20	0.9792	0.9599	0.9803	0.9620
		30	0.9906	0.9815	0.9911	0.9824
	2	10	0.9221	0.8607	0.9285	0.8736
		20	0.9786	0.9585	0.9801	0.9617
		30	0.9903	0.9809	0.9910	0.9823
	3	10	0.9221	0.8608	0.9278	0.8724
		20	0.9785	0.9585	0.9800	0.9614
		30	0.9903	0.9809	0.9909	0.9821
2	0	10	0.9631	0.9379	0.9517	0.9156
		20	0.9893	0.9797	0.9865	0.9740
		30	0.9551	0.9905	0.9938	0.9879
	1	10	0.9491	0.9087	0.9536	0.9189
		20	0.9860	0.9729	0.9870	0.9750
		30	0.9936	0.9875	0.9941	0.9884
	2	10	0.9412	0.8929	0.9526	0.9171
		20	0.9840	0.9689	0.9867	0.9745
		30	0.9928	0.9857	0.9940	0.9881
	3	10	0.9413	0.8936	0.9515	0.9151
		20	0.9840	0.9689	0.9864	0.9738
		30	0.9927	0.9857	0.9938	0.9879

Table 5: Frequency ratios of SS and SC FG nanobeam with $L=20$nm and $m=3$.

Boundary conditions	μ (nm^2)	$q_{\max/r}$ -2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0
SS	2	0.9864	0.9833	0.9800	0.9773	0.9762	0.9773	0.9800	0.9833	0.9864
	4	0.9738	0.9678	0.9614	0.9561	0.9540	0.9561	0.9614	0.9678	0.9738
SC	2	0.9860	0.9811	0.9767	0.9735	0.9724	0.9735	0.9767	0.9811	0.9860
	4	0.9732	0.9637	0.9552	0.9491	0.9469	0.9491	0.9552	0.9637	0.9732

nanobeam. The Galerkin method is employed to reduce the nonlinear partial differential (PDE) equations to a nonlinear second-order ordinary differential equation (ODE). The HPM is used to analyze the present nonlinear ordinary equation. According to obtained results, it is seen that the HPM convergence speed is very fast and has more accurate than traditional perturbation method.

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