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Runge-Kutta Discontinuous Galerkin Method with Front Tracking Method for Solving the Compressible Two-Medium Flow on Unstructured Meshes

Haitian Lu[1], Jun Zhu[2], Chunwu Wang[2] and Ning Zhao[1,*]

[1] *College of Aerospace Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, China*
[2] *College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing, Jiangsu 210016, China*

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Abstract. In this paper, we extend using the Runge-Kutta discontinuous Galerkin method together with the front tracking method to simulate the compressible two-medium flow on unstructured meshes. A Riemann problem is constructed in the normal direction in the material interfacial region, with the goal of obtaining a compact, robust and efficient procedure to track the explicit sharp interface precisely. Extensive numerical tests including the gas-gas and gas-liquid flows are provided to show the proposed methodologies possess the capability of enhancing the resolutions nearby the discontinuities inside of the single medium flow and the interfacial vicinities of the two-medium flow in many occasions.

AMS subject classifications: 65M60, 65M99, 35L65

Key words: Runge-Kutta discontinuous Galerkin method, front tracking method, two-medium flow, Riemann problem, unstructured mesh.

1 Introduction

The algorithms for simulating the compressible two-medium flow are usually consisted of two parts: one is to solve the single-medium flow precisely and the other is to treat the material interface accurately. On the one hand, in recent years, the discontinuous Galerkin (DG) method has been a research hotspot in the simulations of the single medium flow. The original DG method was introduced by Reed and Hill [27] for solving the linear equations in the framework of neutron transport. A major development of

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*Corresponding author.
Email: lhtgkzy@126.com (H. T. Lu), zhujun@nuaa.edu.cn (J. Zhu), wangcw@nuaa.edu.cn (C. W. Wang), nzhao2000@hotmail.com (N. Zhao)

the DG method was carried out by Cockburn and Shu in a series of papers [5–9]. They employed the total variation diminishing (TVD) high order Runge-Kutta time discretization [29] and DG discretization in space with exact or approximate Riemann solvers as interface fluxes and a total variation bounded (TVB) limiter [28] to achieve non-oscillatory property for strong shocks. These methods are termed as Runge-Kutta discontinuous Galerkin (RKDG) methods. On the other hand, a relatively dominant part is the treatment of the moving material interface and its vicinity for simulating compressible two-medium flow. Early algorithms have treated the material interfaces with the γ-based model [17], the mass fraction model [1,18] or a level set function [21,23]. These algorithms, based on shock capturing methods, always yield a numerical diffusion of contact discontinuities over several nodes. However, for the front tracking method [3,12–14], fluid interfaces are explicitly tracked by connected marker points and a sharp interface boundary is maintained during the computation. The ghost fluid method (GFM) introduced by Fedkiw et al. [2,10,11] presents a fairly simple and flexible way to treat multi-medium flows. However, when the pressure or the velocity experiences a large gradient across the interface, the GFM may cause numerical inaccuracy. Indeed, the ghost fluid states should consider the influence of both wave interaction and material properties on the interfacial evolution. This leads to the proposal of improved versions of GFM, for example, the real ghost fluid method (RGFM) [33]. The RGFM predicts the flow states for the real fluid nodes just next to the interface because wave interaction at the interface can propagate upward and downward simultaneously.

So far, there is less work related to a DG method coupled to the GFM [25,34] technique for two-medium flow simulations and the corresponding method to track the interface is almost on structured meshes. In this paper, our major intention is to investigate the performance of the RKDG method combined with the front tracking method to solve for the compressible two-medium flow on unstructured meshes. The RGFM is used to define the interface boundary conditions. Unlike interpolating from the fixed grids to obtain the interface velocity [32], we propose the method based on the Riemann problem constructed in the normal direction of the interface to determine the interface motion, and the corresponding Riemann solutions are also used directly to update the real fluid states in the RGFM. In the earlier works of RGFM [33] based on the uniform structured meshes, the cell center states were usually used as the initial conditions to the Riemann problem. However, it may cause some inaccuracies for the unstructured meshes since the mesh size may vary acutely near the material interface. Due to the explicit tracked interface in this paper, we can easily select the initial conditions to the Riemann problem in the same distances in the normal direction of the interface so that the geometrical influences of the triangular unstructured meshes are avoided.

Except the better adaptability with the complex boundary of the unstructured meshes, the main purposes to use the RKDG method in the simulation of the two-medium flow are in the following: firstly, the higher order accuracy can be obtained in smooth regions easily. Secondly, the initial conditions to the Riemann problem are obtained directly from the solution polynomials in the RKDG method, in contrast to the