Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. 2, No. 1, pp. 118-130

DOI: 10.4208/aamm.09-m0939
February 2010

Travelling Wave Solutions to the Zhiber-Shabat and Related Equations Using Rational Hyperbolic Method

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Received 15 May 2009; Accepted (in revised version) 04 September 2009
Available online 31 December 2009

Abstract. This paper presents the application of a new method for obtaining new exact solutions of some well-known nonlinear partial differential equations. The Rational Hyperbolic method is used for handling the Zhiber-Shabat equation and the related equations such as Liouville, Sinh-Gordon, Dodd-Bullough-Mikhailov and Tzitzeica-Dodd-Bullough equations. We show power of the Rational Hyperbolic method that is simple and effective for solving nonlinear partial differential equations.

AMS subject classifications: 35C07, 35G50

Key words: The rational hyperbolic method, Zhiber-Shabat, Liouville, Sinh-Gordon, Dodd-Bullough-Mikhailov and Tzitzeica-Dodd-Bullough equations.

1 Introduction

Nonlinear evolution equations are widely used as models to describe complex physical phenomena and have a significant role in several scientific and engineering fields. These equations appear in solid state physics [1], fluid mechanics [2], chemical kinetics [3], plasma physics [4], population models, nonlinear optics, propagation of fluxions in Josephson junctions, etc.

Analytical exact solutions to nonlinear partial differential equation play an important role in nonlinear science, since they can provide us much physical information and more insight into the physical aspects of the problem and thus lead to further applications. In recent years, quite a few methods for obtaining explicit travelling and

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solitary wave solutions of nonlinear evolutions equations have been proposed. A variety of powerful methods, such as inverse scattering method [5, 6], bilinear transformation [7], Bucklund and Darboux transformation [7–9], Hamiltonian structures [10], transformed rational [11], the tanh-sech method [12, 13], extended tanh method [14], Exp-Function method [15–18], the sine-cosine method [19–21], the Jacobi elliptic function method [22–24], F-expansion method [25, 26], Lie group analysis [27], He's variational iteration method [28, 29], He's homotopy perturbation method [29–31] and homogeneous balance method [32, 33] and so on.

Nonlinear equations play a major role in scientific fields. A class of equations, namely,

$$u_{xt} + f(u) = 0,$$

play a significant role in many scientific applications such as solid-state physics, nonlinear optics and quantum field theory where the function $f(u)$ takes many forms. In this paper, we investigate the nonlinear Zhiber-Shabat equation in the form

$$u_{xt} + pe^{u} + qe^{-u} + re^{-2u} = 0, \tag{1.1}$$

where p, q, and r are arbitrary constant [34–39]. When q=r=0 and p=1, we have:

$$u_{xt} + e^{u} = 0, \tag{1.2}$$

which is the well-known Liouville equation. When p=1, q=-1 and r=0, we have:

$$u_{xt} + e^{u} - e^{-u} = 0, \tag{1.3}$$

which is the well-known Sinh-Gordon equation. When q=0, p=1 and r=1, we have:

$$u_{xt} + e^{u} + e^{-2u} = 0, \tag{1.4}$$

which is the well-known Dodd-Bullough-Mikhailov equation. Moreover when p=0, q=-1 and r=1, we obtain the Tzitzeica-Dodd-Bullough equation:

$$u_{xt} - e^{-u} + e^{-2u} = 0. \tag{1.5}$$

The aforementioned equations play a significant role in many scientific applications such as solid-state physics, nonlinear optics, plasma physics, fluid dynamics, mathematical biology, nonlinear optics, dislocations in crystals, kink dynamics, and chemical kinetics, and quantum field theory [41–48]. Primarily, we introduce a wave variable η defined as η=$\lambda(x - \alpha t)$, where α is the wave speed. By using the traveling wave transformation $u(x,t)$=$U(\eta)$, we can write the Zhiber-Shabat equation (1.1) in the form

$$-\alpha\lambda^2 U'' + pe^{u} + qe^{-u} + re^{-2u} = 0. \tag{1.6}$$

From Eq. (1.2) we can write the Liouville equation in the form

$$-\alpha\lambda^2 U'' + e^{u} = 0. \tag{1.7}$$