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Optimum Design of Structures for Earthquake Induced Loading by Genetic Algorithm Using Wavelet Transform

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Abstract. Optimum design of structures is achieved by genetic algorithm. The evolutionary algorithm is employed to design structures. The method improves the computing efficiency of the large-scale optimization problems and enhances the global convergence of the design process. The loads are considered as earthquake loads. A time history analysis is carried out for the dynamic analysis. To decrease the computational work, a wavelet transform is used by which the number of points in the earthquake record is reduced. A reverse wavelet transform is also employed to reconstruct the functions under consideration in the time domain. A number of space structures are designed for minimum weight and the results are compared with exact dynamic analysis.

AMS subject classifications: 65T50, 74F99
Key words: Genetic algorithm, discrete wavelet transform, reverse wavelet transform, dynamic analysis.

1 Introduction

Optimum design of structures is to select the design variables systematically such that the weight of the structure is minimized while all the design constraints are satisfied. The external loads can be static [1–3] or dynamic [4–6]. In the present study, the design variables are considered as the member cross-sectional areas, which are chosen from a set of available values (discrete variables). The design constraints are bounds on member stresses and joint displacements. The optimum design problem against earthquake loads is formulated as a mathematical nonlinear programming problem and the solution is obtained by genetic algorithm (GA). The GA method has the capability of finding the global optimal solution while a time history dynamic analysis is employed.

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The probabilistic nature of the standard GA makes the convergence of the method slow. This is due to the fact that the control probabilities for some of the GA operations such as crossover and mutation are chosen constant during the optimization process [7–9]. Another aspect of the GA technique is that the computational cost of the process is high. For problems with large number of degrees of freedom, the structural analysis is time consuming. This makes the optimal design process very inefficient, especially when a time history analysis is considered. To overcome this difficulty, a discrete wavelet transform (DWT) [10–12] is used to transfer the ground acceleration record of the specified earthquake into a function with very small number of points. Thus the time history dynamic analysis is carried out at a fewer points. Another reverse discrete wavelet transform (RWT) is employed to obtain the results of the dynamic analysis for the original earthquake accelerograph record. The numerical results of the dynamic analysis show that this approximation is a powerful technique and the required computational work can be reduced greatly. The error involved in this transformation is small.

In the paper, the details of the optimization approach with approximation concepts will be discussed and some numerical examples for optimum design of structures will be presented. The details of the DWT and RWT will also be outlined. The computational time is compared for the exact optimization method with those of the approximate results.

2 Design problem formulation

The most popular optimization problem in structural design is to minimize the weight. The structure is subjected to constraints imposed on the member stress and joint displacement. This is mathematically shown as:

$$\text{Find } X \text{ to minimize } F(X), \tag{2.1a}$$

$$\text{Subject to } g_j(X) \leq 0, \qquad j = 1, \cdots, m. \tag{2.1b}$$

In this formulation, $X^T = \{x_1, x_2, \cdots, x_n\}$ is the vector of design variables with n variables. In this study X is considered as the cross-sectional areas of the elements. The objective function $F(X)$ is normally taken as the structural weight. The m design constraints imposed on the design problem are shown as inequalities of the form $g_j(X) \leq 0$. To solve the above-mentioned constrained optimization problem by the GA method; first the problem must be converted into an unconstrained optimization problem. There are various methods and a simple method is achieved through exterior penalty function method as follows:

$$\phi(X) = F(X) + r_p \sum_{j=1}^{m} \left\{\max[0, g_j(X)]\right\}^2. \tag{2.2}$$

The scalar r_p is a multiplier and by changing this multiplier and minimizing $\phi(X)$, the minimum of $\phi(X)$, approaches minimum of $F(X)$. Genetic algorithm is based on maximization of a positive unconstrained function. This can be achieved as:

$$\Phi(X) = C - \phi(X), \tag{2.3}$$

where $\Phi(X)$ is the fitness function and C is a positive constant and its value must be greater than the largest value of $\phi(X)$ in a generation to ensure the fitness function to be positive. The multiplier r_p is increased in each generation. This can be achieved by different approaches and the following formula is proposed as [9]

$$r_p = r_1[1 + 0.2(p - 1)], \tag{2.4}$$

where r_1 a given initial is value at first generation and p is the generation number. The penalty value increases gradually until it reaches $4r_1$ and then remains constant for the remaining process.

3 Genetic algorithm

Genetic algorithm is a derivative-free stochastic optimization method based mainly on the concepts of natural selection and evolutionary process. The method was first proposed in 1975 [13] and was extended in [14]. The main feature of GA is that it can be used for both continuous and discrete optimization problems. In addition, because of stochastic nature of the method and using a population of design points in each generation usually gives rise to the global optimum. A genetic algorithm encode each point in the design space into a binary bit string called a chromosome and to each point a fitness function such as Eq. (2.3) is associated. Instead of a single point, GA usually creates a set of points as a population, which is then evolved repeatedly toward a better solution. In each generation, the GA produces a new population using genetic operators such as crossover and mutation. Design points with higher fitness values are more likely to survive and to participate in crossover operations. After a number of generations, design points with better fitness values are obtained. Major components of GA include encoding schemes, fitness evaluations, parent selection, crossover operators and mutation operators [14]. The main steps in the standard GA can be summarized as follows:

1. Initialize a population with randomly generated members and evaluate the fitness value of each individual.
2. (a) Select two members from the population with probabilities proportional to their fitness values.
   (b) Apply crossover with a probability equal to the crossover rate.
   (c) Apply mutation with a probability equal to the mutation rate.
   (d) Repeat (a) to (d) until enough members are generated to form the next generation.
3. Repeat Steps 2 and 3 until a stopping criterion is met.

4 Wavelet transform

In wavelet analysis the use of a fully scalable window solves the signal-cutting problem. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated with varying windows length for every new cycle. The final resolution will be a collection of time-frequency representation of the signal, all with different resolutions. This method is referred to as multiresolution analysis. In wavelet formulation, instead of time-frequency representations, translation-scale representations are used. The scale is employed for the inverse of the frequency. There are two types of wavelet transforms, namely, continues and discrete transforms [15]. In this paper, because of the nature of the earthquake records to reduce the time of analysis, we use a discrete wavelet transform.

5 Discrete wavelet transform

The discrete wavelet transforms (DWT) is defined as follows [15]:

$$DWT(\tau,s)=\sum_{t=0}^{N-1}a(t)\psi^*\Big(\frac{(t-\tau)\delta t}{s}\Big), \tag{5.1}$$

where $DWT(\tau,s)$ is the discrete wavelet transform in two dimensional space τ and s. The symbol $*$ denotes complex conjugate. This equation shows how a function or signal $a(t)$ is decomposed into a set of basis functions ψ. The variables s and τ are scale and translation factors, respectively. N is the number of points in $a(t)$ and δt is the time increment. In Eq. (5.1) ψ is defending as follows:

$$\psi\Big(\frac{(t-\tau)\delta t}{s}\Big)=\Big(\frac{\delta t}{s}\Big)^{0.5}\psi_0\Big(\frac{(t-\tau)\delta t}{s}\Big), \tag{5.2}$$

in which ψ_0 is called the mother wavelets. There are a number of mother wavelets available in wavelet theory. In this study Morlet wavelet [16] is employed as:

$$\psi_0(t)=e^{i\omega_0 t}e^{\frac{-t^2}{2}}. \tag{5.3}$$

Then, we should choose a set of scaling parameters s, such that we adequately sample all of the frequencies present in time series. We first, choose the smallest resolvable scale, s_0, as some multiple of time resolution δt. For the earthquake record δt=0.02 sec is chosen. The smallest wavelet we could possibly resolve is $s_0=b\delta t$, the value of b, is usually greater than one. Then, we chose the larger scales (longer periods or smaller frequencies) as power of two multiples of this smallest scale. In this paper the value and the number of scale s, are chose as [17]:

$$s_j=s_0 2^{J\delta_j},\qquad j=0,1,2,\cdots,J, \tag{5.4}$$

where

$$J = \delta_j^{-1} \log_2 \Big(\frac{N\delta t}{s_0} \Big). \tag{5.5}$$

The largest scale chosen should be less than 0.5 the length of time series [17].

6 Reverse wavelet transform

The wavelet transform is a reversible transform, and the original functions can be recovered from the processed signal. The reconstruction formula [16] can be used by Eq. (6.1).

$$a(t) = \frac{1}{c_\psi} \sum_j \sum_k \frac{1}{s_j^2} DWT(\tau_k, s_j) \psi \Big(\frac{(t-\tau_k)\delta t}{s_j} \Big), \tag{6.1}$$

where c_ψ is a constant value, depends on the wavelet used. The success of the reconstruction depends on this constant called, the admissibility constant, which should satisfy the following condition:

$$c_\psi = 2\pi \int_{-\infty}^{+\infty} \frac{|\widehat{\psi}(\omega)|^2}{|\omega|} d\omega \prec \infty, \tag{6.2}$$

in which $\widehat{\psi}(\omega)$ is the FT of $\psi(t)$. Eq. (6.2) implies that $\widehat{\psi}(0) = 0$, which is:

$$\int_{-\infty}^{+\infty} \psi(t) dt = 0. \tag{6.3}$$

Eq. (6.3) is not a very restrictive requirement since many wavelet functions can be found that the integral is zero. For Eq. (6.3) to be satisfied, the wavelet must be oscillatory.

7 Main steps of optimization with DWT and RWT

The main steps in the optimization process employing DWT and RWT are as follows:

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1. Choose a mother wavelet.
2. Choose a minimum scale s_0, and all other scales.
3. For each scale:
   (a) Choose a location wavelet (τ).
   (b) Calculate Eq. (5.2) at that scale and translation.
   (c) Compute the DWT at that scale and translation by using Eq. (5.1).
   (d) Increase τ, and repeat (a), (b) and (c) until the end of the earthquake record.

4. Carry out the optimization process:
   (a) Analyze the structure with the resolution DWT.
   (b) Use RWT by Eq. (6.1) for calculation of actual response of the structure.
   (c) Check the optimization convergence, if convergence is satisfied, stop, otherwise, go to 4(a).

8 Numerical examples

Two examples are optimized for minimum weight for the El Centro earthquake record (S-E 1940). The time of analysis are computed in CPU time by a personal Pentium 4. The optimization is carried out by the following methods:

(a) Genetic algorithm with exact dynamic analysis (GAE).
(b) Genetic algorithm with DWT and RWT (GAW).

In all the examples, the common features are as follow:

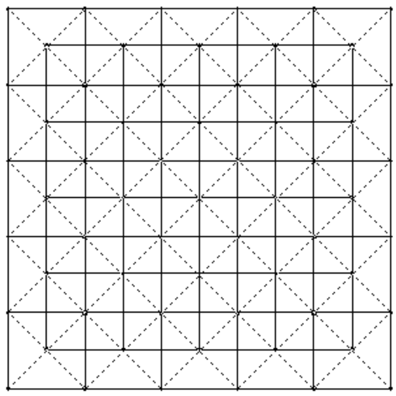
- Allowable stress = 1100kg/cm^2.
- Young's modulus, $E = 2.1 \times 10^6\text{kg/cm}^2$.
- Weight density, $\rho = 0.0078\text{kg/cm}^3$.
- Damping ratio for all modes = 0.05.
- Members are pipe, with radius to thickness less than 50.

8.1 Example 1

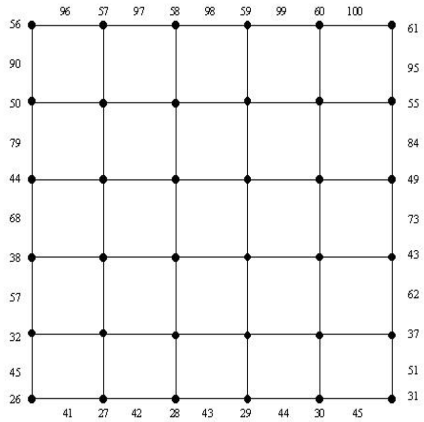
A double layer grid of the type space structure shown in Fig. 1 is chosen with dimensions of 10×10m for top layer and 8×8m for bottom layer. The height of the structure is 0.5m and is simply supported at the corner joints 1, 5, 21 and 25 of the bottom-layer. The mass of $3\text{kg-s}^2/\text{cm}$ is lumped at each free node. The earthquake record is applied in horizontal direction. The problem is designed with stress and vertical displacement constraints. The vertical displacement of joint 13 at the centre of bottom layer must be less than 10cm. The set of available discrete values considered for the cross-sectional areas of the members are given in Table 1. The members are grouped arbitrary into 13 different types as shown in Table 2.

Table 1: Available member areas (cm^2).

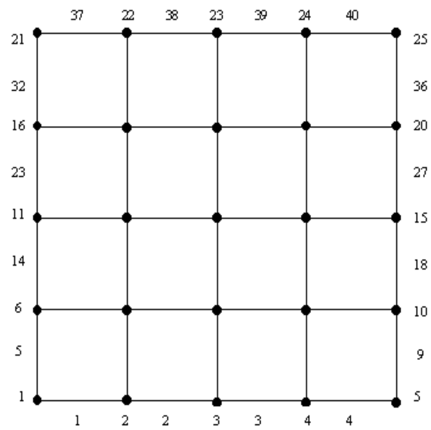
No.	Area	No.	Area	No.	Area	No.	Area	No.	Area	No.	Area
1	0.8272	7	3.267	13	6.563	19	13.66	25	25.11	31	51.03
2	1.127	8	3.493	14	7.413	20	15.11	26	27.54	32	68.35
3	1.427	9	3.789	15	8.229	21	17.13	27	29.69		
4	1.727	10	4.303	16	9.029	22	18.74	28	33.93		
5	2.267	11	4.479	17	10.57	23	19.15	29	40.14		
6	2.777	12	5.693	18	12.99	24	21.15	30	43.02		



(a)



(b)



(c)

Figure 1: (a) Double layer grid. (b) Top layer. (c) Bottom layer.

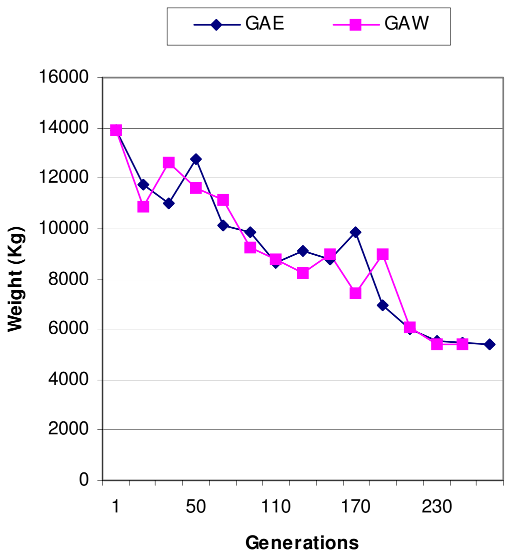


Figure 2: Convergence history for double layer grid with GA.

Table 2: Member grouping for double layer grid.

No.	Member no.	No.	Member no.	No.	Member no.
1	1-4; 37-40	6	7; 16; 25; 34	11	47; 50; 58; 61; 69; 72; 80; 83; 91; 94
2	10-13; 28-31	7	41-45; 96-100	12	48; 49; 59; 60; 70; 71; 81; 82; 92; 93
3	19-22	8	52-56; 85-89	13	All diagonal members
4	5; 9; 14; 18; 23; 27; 32; 36	9	63-67; 74-78		
5	6; 8; 15; 17; 24; 26; 33; 35	10	46; 51; 57; 62; 68; 73; 79; 84; 90; 95		

Table 3: Results of optimization for double layer grid (cm^2).

G	Areas (cm^2)	
	GAE	GAW
1	68.35	68.35
2	10.57	10.57
3	4.303	3.789
4	12.99	12.99
5	10.57	12.99
6	18.74	18.74
7	12.99	10.57
8	12.99	12.99
9	9.029	9.029
10	12.99	12.99
11	4.479	4.479
12	18.74	18.74
13	25.11	25.11
W	5406.8	5389.1
GN	271	221
T	297	37

G is Group no.; W is Wight (kg); GN is Generation no.; T is Time (min.).

Results of optimization, and converge history of the problem for all cases are given in Table 3 and Fig. 2. The number of iterations, in the GAE and GAW is almost the same. In the GAE and GAW the final weights are 5406.8 and 5389.1kg, respectively. The number of required generations in GAE and GAW is about 271 and 221, respectively. The time of computation in GAE and GAW is about 183 and 27min, respectively.

8.2 Example 2

A 72-bar space truss with 48 translation degrees of freedom is shown in Fig. 3. The mass density of material is assumed to be 2.71kg/cm^3 and the mass of 2500kg is lumped at each free node. The earthquake record is applied in X direction. The prob-

Table 4: Member grouping for space truss Example 2.

No.	Member no.	No.	Member no.
1	1, 2, 3, 4, 5, 6	7	37, 38, 39, 40, 41, 42
2	7, 8, 9, 10, 11, 12	8	43, 44, 45, 46, 47, 48
3	13, 14, 15, 16, 17, 18	9	49, 50, 51, 52, 53, 54
4	19, 20, 21, 22, 23, 24	10	55, 56, 57, 58, 59, 60
5	25, 26, 27, 28, 29, 30	11	61, 62, 63, 64, 65, 66
6	31, 32, 33, 34, 35, 36	12	67, 68, 69, 70, 71, 72

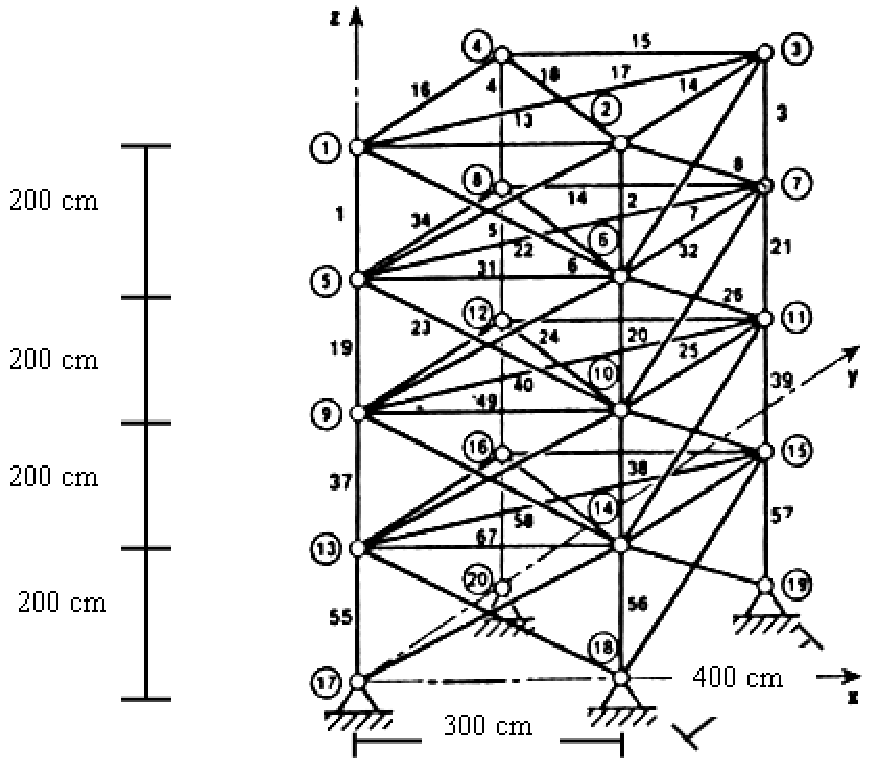


Figure 3: Space truss of Example 2.

lem is designed with stress and horizontal displacement constraints. The horizontal displacement at top joints is considered to be less than 8cm. The set of available discrete values considered for the cross-sectional areas of the members are given in Table 1. The members are grouped arbitrary into 12 different types as shown in Table 4.

Results of optimization and converge history of the problem for all cases are given in Table 5 and Fig. 4. The number of iterations, in the GAE and GAW is almost the same. In the GAE and GAW the final weights are 1382.2 and 1361.7kg, respectively. The number of required generations in GAE and GAW is about 203 and 174, respectively. The time of computation in GAE and GAW is about 116 and 18min, respectively.

Table 5: Results of optimization for Example 2.

G	Areas (cm^2)	
	GAE	GAW
1	0.8272	0.8272
2	1.127	1.127
3	1.727	1.727
4	2.777	3.267
5	10.57	10.57
6	13.66	12.99
7	21.15	19.15
8	27.54	25.11
9	8.229	8.229
10	10.57	10.57
11	10.57	12.99
12	12.99	10.57
W	1382.2	1361.7
GN	203	174
T	156	21

G is Group no.; W is Wight (kg); GN is Generation no.; T is Time (min.).

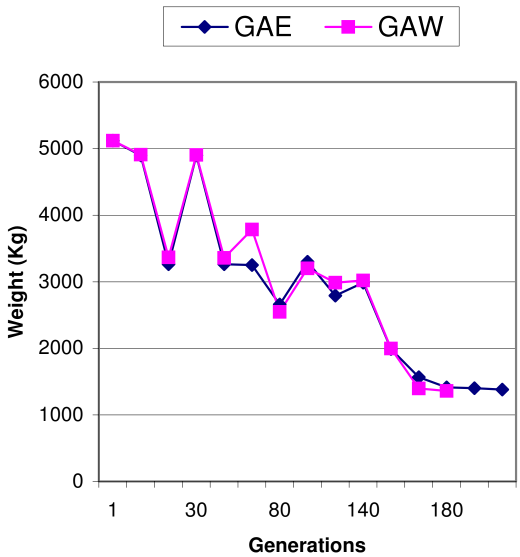


Figure 4: Convergence history for Example 2 with GA.

9 Conclusions

From the numerical results, the following points can be concluded: (1) The number of required generations in GAW is less than GAE; (2) The final objective function in

GAW is less than GAE; (3) Combining GA with wavelet transform reduces the overall optimization cost; (4) Wavelet transform is an effective approach for both dynamic analysis and optimization; (5) The overall time required for optimization is reduced substantially using wavelet transform.

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