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The Pressure-Streamfunction MFS Formulation for the Detection of an Obstacle Immersed in a Two-Dimensional Stokes Flow

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Abstract. In this paper we consider a geometric inverse problem which requires detecting an unknown obstacle such as a submarine or an aquatic mine immersed in a Stokes slow viscous stationary flow of an incompressible fluid, from a single set of Cauchy (fluid velocity and stress force) boundary measurements. The numerical reconstruction is based on the method of fundamental solutions (MFS) for the pressure and streamfunction in two dimensions combined with regularization. Numerical results are presented and discussed.

AMS subject classifications: 33A65, 65N35, 65N22, 65F05, 35J05.

Key words: Stokes flow, method of fundamental solutions, regularization, streamfunction, stress force.

1 Introduction

Recently, the inverse geometric problem of detecting an immersed obstacle in a fluid via non-invasive Cauchy boundary measurements has been addressed in [2]. This inverse problem belongs to the wider class of inverse problems in fluid mechanics, namely, the detection of solid bodies such, as submarines or aquatic mines, from boundary or internal measurements in stationary flows of ideal fluids, see [1], Stokes fluids, see [3, 12, 14, 22, 26], Oseen fluids, see [21], or Navier-Stokes fluids, see [11].

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Prior to this study, Alves and Martins [5] and Martins and Silvestre [22] recently considered the application of the MFS for the detection of immersed obstacles in two-dimensional potential and Stokes flow, respectively. In [22], the authors used the velocity-pressure formulation of the inverse problem given by

$$\Delta \boldsymbol{u} = \nabla p, \qquad \text{in} \quad \Omega \backslash \overline{D}, \tag{1.1a}$$

$$\nabla \cdot \boldsymbol{u} = 0, \qquad \text{in} \quad \Omega \backslash \overline{D}, \tag{1.1b}$$

$$\boldsymbol{u} = \boldsymbol{f}, \qquad \text{on} \quad \partial\Omega, \tag{1.1c}$$

$$\boldsymbol{t} = \boldsymbol{g}, \qquad \text{on} \quad \partial\Omega, \tag{1.1d}$$

$$\boldsymbol{u} = \boldsymbol{0}, \qquad \text{on} \quad \partial D, \tag{1.1e}$$

where $\boldsymbol{u}$ is the fluid velocity, p is the fluid pressure, $\Omega \subset \mathbb{R}^2$ is a bounded domain, $\overline{D} \subset \Omega$ is the unknown obstacle with Lipschitz boundary ∂D such that $\Omega \backslash D$ is connected,

$$\boldsymbol{t} = T\boldsymbol{n}, \tag{1.2}$$

is the stress force,

$$T = \nabla \boldsymbol{u} + (\nabla \boldsymbol{u})^{tr} - pI, \tag{1.3}$$

is the stress tensor, $\boldsymbol{n}$ is the outward unit normal to the boundary, and $\boldsymbol{f}$ and $\boldsymbol{g}$ are given functions satisfying

$$\int_{\partial\Omega} \boldsymbol{f} \cdot \boldsymbol{n}\, ds = 0.$$

In Eq. (1.1e), the no-slip velocity condition can be replaced by the zero stress force (traction) boundary condition

$$\boldsymbol{t} = \boldsymbol{0}, \quad \text{on} \quad \partial D. \tag{1.4}$$

Note that the solution of the inverse problem (1.1) is unique if

$$\boldsymbol{f} \not\equiv \boldsymbol{0},$$

i.e., the unknown obstacle D is identifiable, see [4].

In recent years, the MFS has been widely used for the solution of inverse obstacle detection problems [5, 6, 8, 20, 22] due to the simplicity with which it can be implemented and its rapid convergence properties, especially in three dimensions.

In this study, we reformulate the two-dimensional inverse problem (1.1) in terms of the streamfunction-pressure, see [25]. In this way, in the application of the MFS we use the much simpler fundamental solutions of the Laplace and biharmonic operators instead of the more complicated Stokeslets vectorial fundamental solution, [15]. Remark also that the usual streamfunction-vorticity formulation, [23], is not appropriate since the pressure is present, through the stress force $\boldsymbol{t}$, via (1.3), in the boundary condition (1.1d).

The rest of the numerical procedure for the object identification follows the regularized non-linear least-squares minimization employed in [8] and [20] for cavity identification in electrostatics.