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A Well-Balanced Kinetic Scheme for Gas Dynamic Equations under Gravitational Field

Kun Xu[1,*], Jun Luo[1] and Songze Chen[2]

[1] *Department of Mathematics, Hong Kong University of Science and Technology, Hong Kong*

[2] *College of Engineering, Peking University, Beijing, China*

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Abstract. In this paper, a well-balanced kinetic scheme for the gas dynamic equations under gravitational field is developed. In order to construct such a scheme, the physical process of particles transport through a potential barrier at a cell interface is considered, where the amount of particle penetration and reflection is evaluated according to the incident particle velocity. This work extends the approach of Perthame and Simeoni for the shallow water equations [Calcolo, 38 (2001), pp. 201-231] to the Euler equations under gravitational field. For an isolated system, this scheme is probably the only well-balanced method which can precisely preserve an isothermal steady state solution under time-independent gravitational potential. A few numerical examples are used to validate the above approach.

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Key words: Gas-kinetic scheme, gas dynamic equations, gravitational potential.

1 Introduction

In order to develop an accurate flow solver for a slowly evolving gas dynamic system under gravitational field, the numerical scheme has to be a well-balanced one. For example, most astrophysical problems are related to the hydrodynamical evolution in a gravitational field, a correct implementation of the gravitational force in an astrophysical hydrodynamical code is essential to capture the long time evolution in the modeling star and galaxy formation. Even though many hydrodynamical codes have been successfully applied to astrophysical problems, including the Piecewise Parabolic Method (PPM) and Total Variation Diminishing (TVD) codes [2, 5], most

*Corresponding author.
URL: `http://www.math.ust.hk/~makxu/`
Email: makxu@ust.hk (K. Xu), maluojun@ust.hk (J. Luo), jacksongze@tom.com (S. Chen)

have considered only short time evolutions with strong shock or expansion waves. With the slowness of galaxy evolution, many codes have difficulties due to the improper treatment of the gravitational force effect, the so-called source term in the Euler or the Navier-Stokes equations. A simple example to check the validity of the code is to test for an isolated gas system under time-independent gravitational field. Will the solution settle down to an isothermal steady state solution? Most times, the solution will either oscillate around the equilibrium state, or simply deviate from equilibrium one due to artificial heating, which triggers numerical gravitation-thermal instability, i.e., the collapse of the gas core. There have been many attempts to construct such a well-balanced gas dynamic code that preserves the hydrostatic solution accurately [1,3,9].

In an earlier approach, we have developed an accurate scheme for the Navier-Stokes equations under gravitational filed [6], where the flux function across a cell interface has explicitly taken into account the gravitational forcing on the particle transport. Even though the scheme is very accurate in comparison with operator splitting methods, its inadequate representation of an exact exponential density distribution inside each cell makes it not be a well-balanced one. In this paper, instead of using continuous approximation of a gravitational potential, a constant gravitational potential inside each cell with a potential jump at a cell interface is adopted. With the inclusion of the particle transport mechanism, such as penetration and reflections, a well-balanced gas-kinetic scheme for the gas dynamic equations can be developed. The development of the current method is motivated by the research work of Perthame and Simeoni [4], where the shallow water system was considered. The scheme presented in this paper is probably the only scheme which is a well-balanced one for the Euler equations under the gravitational field. Theoretically, with the piecewise discontinuous potential approximation, a well-balanced BGK-NS scheme can be constructed to solve the Navier-Stokes equations under gravitational field as well with the consideration of particles transport mechanism across a potential jump at a cell interface [8].

2 Kinetic equation for flow system with gravitational source term

In this section, we are going to present the transition from the development of a well-balanced scheme for the shallow water equations to the gas dynamic equations. It is certainly true that for the shallow water equations, there are many well-balanced schemes. The current approach is just one of the successful methods. However, the shallow water equations are much simpler than the gas dynamic equations. For the gas dynamic equation, the method presented here is probably unique in the designing of a well-balanced scheme.

For hydrostatic flows, a proper representation of the hydrostatic balance for the gas dynamic equations is critically important for slowly evolving system under gravitational field. The difficulty associated with this kind of system is the source term