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Long Wavelength Approximation to Peristaltic Motion of Micropolar Fluid with Wall Effects

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Abstract. Peristaltic motion of an incompressible micropolar fluid in a two-dimensional channel with wall effects is studied. Assuming that the wave length of the peristaltic wave is large in comparison to the mean half width of the channel, a perturbation method of solution is obtained in terms of wall slope parameter, under dynamic boundary conditions. Closed form expressions are derived for the stream function and average velocity and the effects of pertinent parameters on these flow variables have been studied. It has been observed that the time average velocity increases numerically with micropolar parameter. Further, the time average velocity also increases with stiffness in the wall.

AMS subject classifications: 76Z05, 76A05.

Key words: Peristaltic motion, micropolar fluid, dynamic boundary conditions.

1 Introduction

The fluid mechanics of peristaltic motion has been extensively studied for several years as it is known to be one of the main mechanism for fluid transport in biological systems. From the point of view of fluid mechanics, peristaltic pumping is characterized by dynamic interaction of fluid flow with the movement of a flexible boundary. In fact peristalsis is the major mechanism for the transport of urine from kidney to bladder, food mixing in the intestines etc. It is also speculated that peristalsis is involved in the vasomotion of small blood vessels. Also mechanical devices like finger pumps and roller pumps use peristalsis to pump blood, slurries, corrosive fluids and so on.

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It is well-known that many physiological fluids behave in general like suspensions of deformable or rigid particles in a Newtonian fluid. Blood, for example, is a suspension of red cells, white cells and platelets in plasma. Another example is cervical mucus, which is a suspension of macromolecules in a water-like liquid. Several investigators have tried to account for the suspension behaviour of biofluids by considering them to be non-Newtonian.

The model of micropolar fluid introduced by Eringen [1] represents a fluid consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformations of the particles are ignored. The main advantage of using micropolar fluid model compared to other non-Newtonian fluids is that it takes care of the rotation of fluid particles by means of an independent kinematic vector called the microrotation vector.

Several authors [2–5] have studied peristaltic transport of a Newtonian fluid in both mechanical and physiological situations under different conditions. Further, peristaltic motion of non-Newtonian fluids also received attention. Shukla et al. [6] studied effect of peristaltic and longitudinal wave motion of the channel wall on movement of microorganisms. Srinivasacharya et al. [7] considered peristaltic transport of a micropolar fluid in a circular tube under low Reynolds number and long wave length approximation. Maruthi Prasad and Radhakrishnamacharya [8] discussed peristaltic transport of a Herschel-Bulkley fluid in a channel in the presence of magnetic field of low intensity. However, all these investigations did not take the wall effects into consideration. Mittra and Prasad [9] considered peristaltic transport in a two-dimensional channel considering the elasticity of the walls. They used dynamic boundary conditions and solved this problem under the approximation of small amplitude ratio. Radhakrishnamacharya and Srinivasulu [10] studied the same problem under long wave length approximation. Muthu et al. [11] extended the analysis of Mittra and Prasad [9] to micropolar fluids. However, no attempt has been made to study the influence of wall properties on peristaltic transport of a micropolar fluid using the dynamic boundary conditions under long wave length approximation.

Hence in the present study, the influence of wall effects on the peristaltic motion of a micropolar fluid in a two-dimensional channel using the dynamic boundary conditions is investigated. Perturbation method of solution has been obtained in terms of wall slope parameter assuming that the wave length of the peristaltic wave is large in comparison to the mean half width of the channel. Expressions for the stream function and average velocity have been derived and the effects of various parameters on these flow variables have been studied.

2 Formulation of the problem

We consider the flow of an unsteady incompressible micropolar fluid through a two dimensional channel of width $2d$ and with flexible walls on which are imposed traveling sinusoidal waves of long wave length. Cartesian coordinate system (x, y) is chosen with the x-axis aligned with the centre line of the channel. The traveling waves are rep-

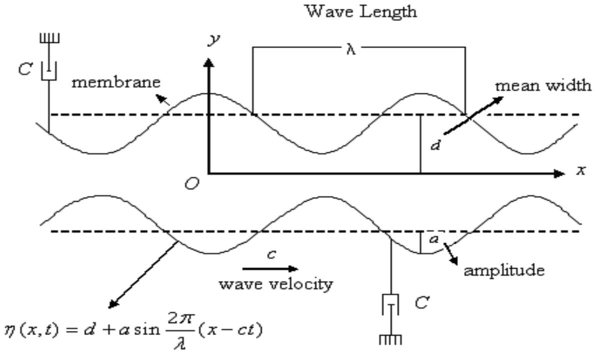

Figure 1: Geometry of a two-dimensional peristaltic motion of the walls.

resented by (Fig. 1) $a \sin 2\pi(x-ct)/\lambda$, where a is the amplitude, λ is the wave length and c is the wave speed of the traveling waves. The equations governing the peristaltic motion of incompressible micropolar fluid for the present problem are given [11] as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\left(\frac{2\mu+\kappa}{2}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial g}{\partial y}, \tag{2.2}$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\left(\frac{2\mu+\kappa}{2}\right)\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\kappa\frac{\partial g}{\partial x}, \tag{2.3}$$

$$\rho J\left(\frac{\partial g}{\partial t}+u\frac{\partial g}{\partial x}+v\frac{\partial g}{\partial y}\right)=-2\kappa g+\gamma\left(\frac{\partial^2 g}{\partial x^2}+\frac{\partial^2 g}{\partial y^2}\right)+\kappa\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right), \tag{2.4}$$

where $u(x,y,t)$ and $v(x,y,t)$ are the velocity components in the x- and y- directions respectively, $g(x,y,t)$ is the microrotation component in the direction normal to the both x and y axes. Here J is the microinertia constant, μ is the viscosity coefficient of classical fluid dynamics, κ and γ are the new viscosity coefficients for the micropolar fluids, ρ is the density of the fluid.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero.

Thus the no-slip boundary conditions for the velocity and microrotation are

$$u=0,\quad g=0,\quad \text{at}\quad y=\pm\eta=\pm\Big[d+a\sin\frac{2\pi}{\lambda}\,(x-ct)\Big]. \tag{2.5}$$

The dynamic boundary conditions at the flexible walls, following Mittra and Prasad [9], can be written as

$$\frac{\partial L(\eta)}{\partial x}=-\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)+\left(\frac{2\mu+\kappa}{2}\right)\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\kappa\frac{\partial g}{\partial y},$$

at $y=\pm\eta(x,t)$, where

$$\frac{\partial L(\eta)}{\partial x}=\frac{\partial p}{\partial x}=-T\frac{\partial^3\eta}{\partial x^3}+m\frac{\partial^3\eta}{\partial t^2\partial x}+C\frac{\partial^2\eta}{\partial t\partial x}, \tag{2.6}$$

Here T is the tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping force.

We define the stream function ψ by

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}, \tag{2.7}$$

and eliminating the pressure between (2.2) and (2.3), Eqs. (2.2)-(2.4), become

$$\begin{aligned}&\rho\left[\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi+\frac{\partial\psi}{\partial y}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\partial\psi}{\partial x}-\frac{\partial\psi}{\partial x}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\frac{\partial\psi}{\partial y}\right]\\ =&\left(\frac{2\mu+\kappa}{2}\right)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\psi+\kappa\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g, \end{aligned}\tag{2.8}$$

$$\rho J\left(\frac{\partial g}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial g}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial g}{\partial y}\right)=\gamma\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g-\kappa\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi+2g\right]. \tag{2.9}$$

Introducing the following non-dimensional quantities

$$x'=\frac{x}{\lambda},\quad y'=\frac{y}{d},\quad t'=\frac{tc}{\lambda},\quad \psi'=\frac{\psi}{cd},\quad \eta'=\frac{\eta}{d},\quad g'=\frac{gd}{c}, \tag{2.10}$$

Eqs. (2.8), (2.9), (2.5) and (2.6), after dropping the primes, can be written as

$$\begin{aligned}&\delta R_e\left[\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_t+\psi_y\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_x-\psi_x\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_y\right]\\ =&\left(\frac{2+\mu_1}{2}\right)\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2\psi+\mu_1\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g,\end{aligned} \tag{2.11}$$

$$\begin{aligned}&\delta R_l\left(\frac{\partial g}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial g}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial g}{\partial y}\right)\\ =&2(1-N^2)\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)g-N^2M^2\left[\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi+2g\right].\end{aligned} \tag{2.12}$$

The boundary conditions are

$$\psi_y=0,\qquad g=0,\qquad \text{at}\qquad y=\pm\eta=\pm\big[1+\varepsilon\sin 2\pi(x-t)\big], \tag{2.13}$$

$$\begin{aligned}&-\delta\left(\frac{\partial^2\psi}{\partial y\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)+\left(\frac{2+\mu_1}{2R_e}\right)\left(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi_y\\ &+\frac{\mu_1}{R_e}\frac{\partial g}{\partial y}=E_1\frac{\partial^3\eta}{\partial x^3}+E_2\frac{\partial^3\eta}{\partial t^2\partial x}+E_3\frac{\partial^2\eta}{\partial t\partial x},\end{aligned} \tag{2.14}$$

where $\varepsilon(=a/d)$, $\delta(=d/\lambda)$ are geometric parameters, $R_e(=\rho cd/\mu)$ is the Reynolds number, $\mu_1=\kappa/\mu$ denotes non-dimensional quantity for micropolar fluid and

$$\begin{aligned}&N=\sqrt{\frac{\mu_1}{2+\mu_1}}, && M=2d\sqrt{\frac{\mu}{\gamma}}, && K_2=\frac{Cd^3}{\mu\lambda^2},\\ &K_3=\frac{Td^4}{\mu^2\lambda^3}, && m_1=\frac{md^2}{\rho\lambda^3}, && R_l=\frac{4\rho cd\mu J}{\gamma\,(2\mu+\kappa)},\\ &E_1=-\frac{K_3}{R_e^2}, && E_2=m_1, && E_3=\frac{K_2}{R_e}.\end{aligned}$$

Boundary condition (2.13) is the classical no slip boundary condition. The boundary condition (2.14) is the dynamic boundary condition at the flexible walls, see, e.g., [9-11].

The parameters μ_1 and M are non-dimensional quantities characterising micropolar fluid. Also μ_1, which characterizes the coupling of (2.11) and (2.12), denotes the ratio of the viscosity coefficient for the microploar fluids and classical viscosity coefficient. The parameter M can be thought of as a fluid property depending upon the size of microstructure. It can be seen that when κ and γ are zero, that is, when μ_1 becomes zero and M tends to infinity, (2.11) and (2.12) reduce to the classical Navier-Stokes equations. It is reasonable to assume that the modified Reynolds number R_l, which involves the square of a length of typical microstructure J, is much less than unity.

The parameter E_1 characterizes the rigiditive nature of the wall and depends on the wall tension. Further, E_2 and E_3 represent respectively the stiffness and the dissipative features of the walls. Also, $E_3 = 0$ implies that the walls moves up and down with no damping force on them and hence indicates the case of elastic walls.

3 Method of solution

We seek perturbation solution in terms of small parameter δ as follows:

$$F = F_0 + \delta F_1 + \delta^2 F_2 + \cdots, \tag{3.1}$$

where F represents any flow variable.

Substituting (3.1) in (2.11) to (2.14) and collecting the coefficients of various powers of δ, we get the following sets of equations:

Zeroth Order:

$$\Big(\frac{2+\mu_1}{2}\Big)\frac{\partial^4\psi_0}{\partial y^4} + \mu_1\frac{\partial^2 g_0}{\partial y^2} = 0, \tag{3.2}$$

$$2(1-N^2)\frac{\partial^2 g_0}{\partial y^2} - N^2M^2\Big(\frac{\partial^2\psi_0}{\partial y^2} + 2g_0\Big) = 0. \tag{3.3}$$

Boundary Conditions:

$$\psi_{0y} = 0, \quad g_0 = 0, \quad \text{at} \quad y = \pm\eta(x,t), \tag{3.4}$$

where

$$\begin{aligned} &\eta(x,t) = 1 + \varepsilon\sin 2\pi(x-t), \\ &\Big(\frac{2+\mu_1}{2R_e}\Big)\psi_{0yyy} + \frac{\mu_1}{R_e}g_{0y} = E_1\frac{\partial^3\eta}{\partial x^3} + E_2\frac{\partial^3\eta}{\partial t^2\partial x} + E_3\frac{\partial^2\eta}{\partial t\partial x}. \end{aligned} \tag{3.5}$$

First order:

$$\Big(\frac{2+\mu_1}{2}\Big)\frac{\partial^4\psi_1}{\partial y^4} + \mu_1\frac{\partial^2 g_1}{\partial y^2} = R_e\Big(\frac{\partial^2}{\partial y^2}\psi_{0t} + \psi_{0y}\frac{\partial^2}{\partial y^2}\psi_{0x} - \psi_{0x}\frac{\partial^2}{\partial y^2}\psi_{0y}\Big), \tag{3.6}$$

$$2(1-N^2)\frac{\partial^2 g_1}{\partial y^2} - N^2M^2\Big(\frac{\partial^2\psi_1}{\partial y^2} + 2g_1\Big) = R_l(g_{0t} + \psi_{0y}g_{0x} - \psi_{0x}g_{0y}). \tag{3.7}$$

Boundary Conditions:

$$\psi_{1y}=0,\quad g_1=0,\quad \text{at}\quad y=\pm\eta(x,t), \tag{3.8}$$

where

$$\begin{aligned}&\eta\,(x,t)=1+\varepsilon\sin 2\pi(x-t),\\ &-(\psi_{0ty}+\psi_{0y}\psi_{0xy}-\psi_{0x}\psi_{0yy})+\Big(\frac{2+\mu_1}{2R_e}\Big)\psi_{1yyy}+\frac{\mu_1}{R_e}g_{1y}=0.\end{aligned} \tag{3.9}$$

Solving Eqs. (3.2), (3.3), (3.6) and (3.7) under the boundary conditions (3.4), (3.5), (3.8) and (3.9), we finally get:

$$\psi_0=\frac{1}{1-N^2}\Big(\frac{A_1y^3}{6}\Big)-A_3y-\frac{N^2}{1-N^2}\Big(\frac{A_1\sinh NMy}{N^2M^2\sinh NM\eta}\Big), \tag{3.10}$$

$$g_0=A_1\Big(\frac{\eta\sinh NMy-y\sinh NM\eta}{2(1-N^2)\sinh NM\eta}\Big), \tag{3.11}$$

$$\begin{aligned}\psi_1=&-2N^2I+R_m\Big\{\frac{1}{70}A\frac{\partial A}{\partial x}y^7+K_{11}\frac{y^5}{20}+K_{12}\Big(\frac{\sinh NMy}{N^2M^2}\Big)+K_{13}\Big(\frac{y^2\sinh NMy}{N^2M^2}\\ &-\frac{4y\cosh NMy}{N^3M^3}+\frac{6\sinh NMy}{N^4M^4}\Big)-K_{14}\Big(\frac{y\cosh NMy}{N^2M^2}-\frac{2\sinh NMy}{N^3M^3}\Big)\\ &-NML_1\frac{\partial A}{\partial x}\Big(\frac{y^3\cosh NMy}{N^2M^2}-\frac{6y^2\sinh NMy}{N^3M^3}+\frac{18y\cosh NMy}{N^4M^4}\\ &-\frac{24\sinh NMy}{N^5M^5}\Big)+B_1\frac{y^3}{6}+B_5y,\end{aligned} \tag{3.12}$$

$$\begin{aligned}g_1=&P_1\Big(y-\frac{\eta\sinh NMy}{\sinh NM\eta}\Big)+P_3\Big(y^3-\frac{\eta^3\sinh NMy}{\sinh NM\eta}\Big)+P_5\Big(y^5-\frac{\eta^5\sinh NMy}{\sinh NM\eta}\Big)\\ &+\frac{b_{11}}{2N^2M^2}(y\cosh NMy-\coth NM\eta\sinh NMy)+\frac{b_{12}}{4NM}\Big\{\frac{2}{3}y^3\cosh NMy\\ &-\frac{y^2\sinh NMy}{NM}+\frac{y\cosh NMy}{N^2M^2}-\Big(\frac{2}{3}\eta^3\cosh NM\eta-\frac{\eta^2\sinh NM\eta}{NM}\\ &+\frac{\eta\cosh NM\eta}{N^2M^2}\Big)\frac{\sinh NMy}{\sinh NM\eta}\Big\}+\frac{b_{13}}{4NM}\Big\{y^2\sinh NMy-\frac{y\cosh NMy}{NM}\\ &-\Big(\eta^2\sinh NM\eta-\frac{\eta\cosh NM\eta}{NM}\Big)\frac{\sinh NMy}{\sinh NM\eta}\Big\}+\frac{b_{14}}{4NM}\Big\{\frac{1}{2}y^4\sinh NMy\\ &-\frac{y^3\cosh NMy}{NM}+\frac{3y^2\sinh NMy}{2N^2M^2}-\frac{3y\cosh NMy}{2N^3M^3}-\Big(\frac{\eta^4\sinh NM\eta}{2}\\ &-\frac{\eta^3\cosh NM\eta}{NM}+\frac{3\eta^2\sinh NM\eta}{2N^2M^2}-\frac{3\eta\cosh NM\eta}{2N^3M^3}\Big)\frac{\sinh NMy}{\sinh NM\eta}\Big\}\\ &+\frac{R_n}{6NM}K_{15}\big[\sinh 2NMy-2\sinh NMy\cosh NM\eta\big],\end{aligned} \tag{3.13}$$

$$\begin{aligned}I=&P_1\Big(\frac{y^3}{6}-\frac{\eta\sinh NMy}{N^2M^2\sinh NM\eta}\Big)+P_2\Big(\frac{y^5}{20}-\frac{\eta^3\sinh NMy}{N^2M^2\sinh NM\eta}\Big)\\ &+P_3\Big(\frac{y^7}{42}-\frac{\eta^5\sinh NMy}{N^2M^2\sinh NM\eta}\Big)+\frac{b_{11}}{2N^2M^2}\Big(\frac{y\cosh NMy}{N^2M^2}-\frac{2\sinh NMy}{N^3M^3}\end{aligned}$$

$$
\begin{aligned}
&-\frac{\coth NM\eta \sinh NMy}{N^2M^2}\Big) + \frac{b_{12}}{4NM}\Big\{\frac{2y^3\cosh NMy}{3N^2M^2} - \frac{5y^2\sinh NMy}{N^3M^3} \\
&+\frac{17y\cosh NMy}{N^4M^4} - \frac{24\sinh NMy}{N^5M^5} - \frac{2}{3}\Big(\eta^3\cosh NM\eta - \frac{\eta^2\sinh NM\eta}{NM} + \frac{\eta\cosh NM\eta}{N^2M^2}\Big) \\
&\times\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\Big\} + \frac{b_{13}}{4NM}\Big\{\frac{y^2\sinh NMy}{N^2M^2} - \frac{5y\cosh NMY}{N^3M^3} + \frac{8\sinh NMy}{N^4M^4} \\
&-\Big(\eta^2\sinh NM\eta - \frac{\eta\cosh NM\eta}{NM}\Big)\times\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\Big\} + \frac{b_{14}}{4NM}\Big\{\frac{y^4\sinh NMy}{2N^2M^2} \\
&-\frac{5y^3\cosh NMy}{N^3M^3} + \frac{27y^2\sinh NMy}{N^4M^4} - \frac{153y\cosh NMy}{2N^5M^5} + \frac{99\sinh NMy}{N^6M^6} \\
&-\Big(\frac{\eta^4\sinh NM\eta}{2} - \frac{\eta^3\cosh NM\eta}{NM} + \frac{3\eta^2\sinh NM\eta}{2N^2M^2} - \frac{3\eta\cosh NM\eta}{2N^3M^3}\Big) \\
&\times\frac{\sinh NMy}{N^2M^2\sinh NM\eta}\Big\} + \frac{R_n}{6N^2M^2}K_{15}\Big(\frac{\sinh 2NMy}{4NM} - \frac{2\cosh NM\eta\sinh NMy}{NM}\Big),
\end{aligned}
$$

$$
\begin{aligned}
A_1 &= -\frac{2R_e\varepsilon}{2+\mu_1}\big[(2\pi)^3\cos 2\pi(x-t)(E_1+E_2) - (2\pi)^2E_3\sin 2\pi(x-t)\big],\\
A_3 &= \Big(\frac{A_1}{1-N^2}\Big)\Big(\frac{\eta^2}{2} - \frac{N\eta}{M}\coth NM\eta\Big), \qquad A = \frac{A_1}{6(1-N^2)},\\
L_1 &= -\frac{N^2A_1\eta}{(1-N^2)N^2M^2\sinh NM\eta},\\
B_3 &= \frac{A_1\eta}{2(1-N^2)\sinh NM\eta}, \qquad B_4 = -\frac{A_1}{2(1-N^2)},\\
R_m &= \frac{2R_e}{2+\mu_1}, \qquad R_n = \frac{R_l}{2(1-N^2)},\\
P_1 &= \frac{1}{N^2M^2}\Big[-\frac{6R_m}{1-N^2}\Big(\frac{6A}{N^2M^2}\frac{\partial A}{\partial x} + \frac{1}{2}K_{16} - \frac{R_n}{N^2M^2}K_{17}\Big) - R_nK_{17}\Big],\\
P_3 &= -\frac{6R_m}{1-N^2}\Big(\frac{6A}{N^2M^2}\frac{\partial A}{\partial x} + \frac{1}{2}K_{11} - \frac{R_n}{N^2M^2}K_{17}\Big),\\
P_5 &= -\frac{3R_mA}{10(1-N^2)}\frac{\partial A}{\partial x},\\
b_{11} &= \frac{N^2M^2R_m}{2(1-N^2)}K_{12} + R_nK_{18}, \qquad b_{12} = \frac{N^2M^2R_m}{2(1-N^2)}K_{13} + 3R_nA\frac{\partial B_3}{\partial x},\\
b_{13} &= -\frac{N^2M^2R_m}{2(1-N^2)}K_{14} + R_nNMK_{19}, \qquad b_{14} = -\Big(\frac{N^3M^3R_m}{2(1-N^2)}L_1 + R_nB_3NM\Big)\frac{\partial A}{\partial x},\\
K_{11} &= \Big(\frac{\partial A}{\partial t} + A_3\frac{\partial A}{\partial x} - A\frac{\partial A_3}{\partial x}\Big),\\
K_{12} &= \Big(\frac{\partial L_1}{\partial t} + A_3\frac{\partial L_1}{\partial x} + \frac{12A}{N^2M^2}\frac{\partial L_1}{\partial x} + \frac{12L_1}{N^2M^2}\frac{\partial A}{\partial x} + 2L_1\frac{\partial A_3}{\partial x}\Big),\\
K_{13} &= \Big(3A\frac{\partial L_1}{\partial x} + 6L_1\frac{\partial A}{\partial x}\Big), \qquad K_{14} = \Big(\frac{6A}{NM}\frac{\partial L_1}{\partial x} + \frac{18L_1}{NM}\frac{\partial A}{\partial x} + NML_1\frac{\partial A_3}{\partial x}\Big),\\
K_{15} &= \Big(L_1\frac{\partial B_3}{\partial x} - B_3\frac{\partial L_1}{\partial x}\Big), \qquad K_{16} = \Big(\frac{\partial A}{\partial t} + A_3\frac{\partial A}{\partial x} - A\frac{\partial A_3}{\partial x}\Big),
\end{aligned}
$$

$$K_{17} = \Big(3A\frac{\partial B_3}{\partial x} - B_4\frac{\partial A}{\partial x}\Big), \qquad K_{18} = \Big(\frac{\partial B_4}{\partial t} + A_3\frac{\partial B_4}{\partial x} - B_4\frac{\partial L_1}{\partial x}\Big),$$
$$K_{19} = \Big(L_1\frac{\partial B_4}{\partial x} - B_3\frac{\partial A_3}{\partial x}\Big), \qquad K_{10} = \Big(\frac{\partial L_1}{\partial t} + A_3\frac{\partial L_1}{\partial x} + L_1\frac{\partial A_3}{\partial x}\Big),$$
$$K_{20} = \Big(A\frac{\partial L_1}{\partial x} + L_1\frac{\partial A}{\partial x}\Big), \qquad F = \Big(\frac{\mu_1}{R_e} - 2N^2\Big)\Big(\frac{1-\eta NM\coth NM\eta}{2(1-N^2)}\Big) - \Big(\frac{2+\mu_1}{R_e}\Big).$$

Using (2.7), (3.10) and (3.12), we get the expressions for velocity as

$$u_0 = \frac{A_1}{2(1-N^2)}(y^2-\eta^2) + \frac{A_1\eta N^2}{NM(1-N^2)}\Big(\frac{\cosh NM\eta - \cosh NMy}{\sinh NM\eta}\Big), \tag{3.14}$$

$$\begin{aligned} u_1 = &-2N^2G + R_m\Big\{\frac{1}{10}A\frac{\partial A}{\partial x}y^6 + K_{11}\frac{y^4}{4} + K_{12}\frac{\cosh NMy}{NM} + K_{13}\Big(\frac{y^2\cosh NMy}{NM} \\ &- \frac{2y\sinh NMy}{N^2M^2} + \frac{2\cosh NMy}{N^3M^3}\Big) - K_{14}\Big(\frac{y\sinh NMy}{NM} - \frac{\cosh NMy}{N^2M^2}\Big) \\ &- NML_1\frac{\partial A}{\partial x}\Big(\frac{y^3\sinh NMy}{NM} - \frac{3y^2\cosh NMY}{N^2M^2} + \frac{6y\sinh NMy}{N^3M^3} \\ &- \frac{6\cosh NMy}{N^4M^4}\Big)\Big\} + B_1\frac{y^2}{2} + B_5, \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} G &= \frac{dI}{dy}, \\ B_1 &= \frac{1}{d}\Big\{\Big(\frac{\mu_1}{R_e} - 2N^2\Big)(t_1+t_2+t_3+t_4+t_5+t_6) - (t_7+t_8+t_9-t_{10}) \\ &\qquad + \frac{(2+\mu_1)R_m}{R_e}(t_{11}+t_{12})\Big\}, \\ B_5 &= 2N^2(m_1+t_2+t_3+t_4+t_5+m_2) - R_m(m_3+m_4-m_5-m_6) + m_7, \\ d &= \Big(\frac{\mu_1}{R_e} - 2N^2\Big)\Big(\frac{1-\eta NM\coth Nm\eta}{2(1-N^2)}\Big) - \Big(\frac{2+\mu_1}{R_e}\Big), \\ t_1 &= p_1(1-\eta NM\coth NM\eta) + p_3(3\eta^2 - \eta^3 NM\coth NM\eta) \\ &\qquad + p_5(5\eta^4 - \eta^5 NM\coth NM\eta), \\ t_2 &= \frac{b_{11}}{2N^2M^2}(NM\eta\sinh NM\eta + \cosh NM\eta - NM\coth NM\eta\cosh NM\eta), \\ t_3 &= \frac{b_{12}}{6NM}\Big(3\eta^2\cosh NM\eta + NM\eta^3\sinh NM\eta\Big) - \frac{b_{12}}{4N^2M^2}\Big(2\eta\sinh NM\eta \\ &\quad + NM\eta^2\cosh NM\eta\Big) + \frac{b_{12}}{4N^3M^3}(\cosh NM\eta + NM\eta\sinh hNM\eta) \\ &\quad - \frac{b_{12}}{4NM}\Big(\frac{2}{3}\eta^3\cosh NM\eta - \frac{1}{NM}\eta^2\sinh NM\eta + \frac{1}{N^2M^2}\eta\cosh NM\eta\Big)NM\coth NM\eta, \\ t_4 &= \frac{b_{13}}{4NM}(2\eta\sinh NM\eta + NM\eta^2\cosh NM\eta) - \frac{b_{13}}{4N^2M^2}(\cosh NM\eta + NM\eta\sinh NM\eta) \\ &\quad - \frac{b_{13}}{4NM}\Big(\eta^2\sinh NM\eta - \frac{\eta}{NM}\cosh NM\eta\Big)NM\coth NM\eta, \end{aligned}$$

$$\begin{aligned}
t_5 =& \frac{b_{14}}{8NM}(4\eta^3 \sinh NM\eta + NM\eta^4 \cosh NM\eta) - \frac{b_{14}}{4N^2M^2}(3\eta^2 \cosh NM\eta \\
&+ NM\eta^3 \sinh NM\eta) + \frac{3b_{14}}{8N^3M^3}(NM\eta^2 \cosh NM\eta + 2\eta \sinh NM\eta) - \frac{3b_{14}}{8N^4M^4} \\
&\cdot (\cosh NM\eta + NM\eta \sinh NM\eta) - \frac{b_{14}}{4NM}\Big(\frac{1}{2}\eta^4 \sinh NM\eta - \frac{1}{NM}\eta^3 \cosh NM\eta \\
&+ \frac{3}{2N^2M^2}\eta^2 \sinh NM\eta - \frac{3}{2N^3M^3}\eta \cosh NM\eta\Big) NM \coth NM\eta, \\
t_6 =& \frac{R_n}{3}\Big(L_1\frac{\partial B_3}{\partial x} - B_3\frac{\partial L_1}{\partial x}\Big)(\cosh 2NM\eta - \cosh^2 NM), \\
t_7 =& 3\frac{\partial A}{\partial t}\eta^2 + \frac{\partial A_3}{\partial t} + NM\frac{\partial L_1}{\partial t}\cosh NM\eta + 9A\frac{\partial A}{\partial x}\eta^4 + 3A\frac{\partial A_3}{\partial x}\eta^2 \\
&+ 3NMA\frac{\partial L_1}{\partial x}\eta^2 \cosh NM\eta + 3A_3\frac{\partial A}{\partial x}\eta^2, \\
t_8 =& A_3\frac{\partial A_3}{\partial x} + NMA_3\frac{\partial L_1}{\partial x}\cosh NM\eta + 3L_1 NM\frac{\partial A}{\partial x}\eta^2 \cosh NM\eta \\
&+ L_1 NM\frac{\partial A_3}{\partial x}\cosh NM\eta, \\
t_9 =& L_1N^2M^2\frac{\partial L_1}{\partial x}\cosh^2 NM\eta - 6A\frac{\partial A}{\partial x}\eta^4 - L_1N^2M^2\frac{\partial A}{\partial x}\eta^3 \sinh NM\eta - 6A\frac{\partial A_3}{\partial x}\eta^2, \\
t_{10} =& L_1N^2M^2\frac{\partial A_3}{\partial x}\eta \sinh NM\eta + 6A\frac{\partial L_1}{\partial x}\eta \sinh NM\eta + L_1N^2M^2\frac{\partial L_1}{\partial x}\sinh^2 NM\eta, \\
t_{11} =& 3A\frac{\partial A}{\partial x}\eta^4 + 3\Big(\frac{\partial A}{\partial t} + A_3\frac{\partial A}{\partial x} - A\frac{\partial A_3}{\partial x}\Big)\eta^2 + \Big(\frac{\partial L_1}{\partial t} + A_3\frac{\partial L_1}{\partial x} \\
&+ L_1\frac{\partial A_3}{\partial x}\Big) NM \cosh NM\eta, \\
t_{12} =& 3NM\Big(A\frac{\partial L_1}{\partial x} + L_1\frac{\partial A}{\partial x}\Big)\eta^2 \cosh NM\eta - \Big(6A\frac{\partial L_1}{\partial x} + N^2M^2L_1\frac{\partial A_3}{\partial x}\Big)\eta \sinh NM\eta \\
&- N^2M^2L_1\frac{\partial A}{\partial x}\eta^3 \sinh NM\eta, \\
m_1 =& p_1\Big(\frac{\eta^2}{2} - \frac{\eta}{NM}\coth NM\eta\Big) + p_3\Big(\frac{\eta^4}{4} - \frac{\eta^3}{NM}\coth NM\eta\Big) \\
&+ p_5\Big(\frac{\eta^6}{6} - \frac{\eta^5}{NM}\coth NM\eta\Big), \\
m_2 =& \frac{R_n}{6NM}\Big(L_1\frac{\partial B_3}{\partial x} - B_3\frac{\partial L_1}{\partial x}\Big)\Big(\frac{\cosh 2NM\eta}{2NM} - \frac{2\cosh^2 NM\eta}{NM}\Big), \\
m_3 =& \frac{1}{10}A\frac{\partial A}{\partial x}\eta^6 + \Big(\frac{\partial A}{\partial t} + A_3\frac{\partial A}{\partial x} - A\frac{\partial A_3}{\partial x}\Big)\frac{\eta^4}{4} + \Big(\frac{\partial L_1}{\partial t} + A_3\frac{\partial L_1}{\partial x} \\
&+ \frac{12A}{N^2M^2}\frac{\partial L_1}{\partial x} + \frac{12L_1}{N^2M^2}\frac{\partial A}{\partial x} + 2L_1\frac{\partial A_3}{\partial x}\Big)\frac{\cosh NM\eta}{NM}, \\
m_4 =& \Big(3A\frac{\partial L_1}{\partial x} + 6L_1\frac{\partial A}{\partial x}\Big)\Big(\frac{\eta^2 \cosh NM\eta}{NM} - \frac{2\eta \sinh NM\eta}{N^2M^2} + \frac{2\cosh NM\eta}{N^3M^3}\Big), \\
m_5 =& \Big(\frac{6A}{NM}\frac{\partial L_1}{\partial x} + \frac{18L_1}{NM}\frac{\partial A}{\partial x} + NML_1\frac{\partial A_3}{\partial x}\Big)\Big(\frac{\eta \sinh NM\eta}{NM} - \frac{\cosh NM\eta}{N^2M^2}\Big), \\
m_6 =& NML_1\frac{\partial A}{\partial x}\Big(\frac{\eta^3 \sinh NM\eta}{NM} - \frac{3\eta^2 \cosh NM\eta}{N^2M^2} + \frac{6\eta \sinh NM\eta}{N^3M^3} - \frac{6\cosh NM\eta}{N^4M^4}\Big), \\
m_7 =& \Big[\frac{N^2}{1-N^2}\Big(1 - \frac{\eta \coth NM\eta}{NM}\Big) - \frac{\eta^2}{2}\Big]B_1\,.
\end{aligned}$$

The average velocity $\bar{u}$, over one period of the motion is given by

$$\bar{u} = \int_0^1 u dt = \bar{u}_0 + \delta \bar{u}_1 + \cdots . \tag{3.16}$$

4 Results and discussion

The analytical expression for the time average velocity $\bar{u}$ is given by Eq. (3.16). To study the effects of various parameters on the time mean flow $\bar{u}$ has been numerically evaluated using Mathematica software and the results are graphically presented in Figs. 2-13.

The average velocity for the present problem depends upon the following important non-dimensional quantities:

I) The cross viscosity parameter μ_1 which denotes the ratio of the viscosity coefficient for the micropolar fluid and the classical viscous fluid.

II) The micropolar parameter M which characterizes the couple stress effects, due to its dependence on the coefficient γ.

III) E_1, E_2 and E_3, the wall parameters which characterise the viscoelastic behaviour of the flexible walls.

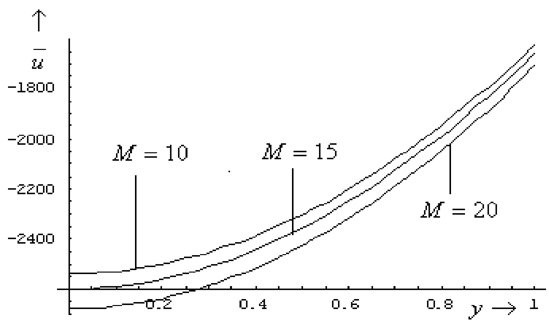

Figure 2: Effect of M on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 4$, $E_3 = 0.06$, $\mu_1 = 0.02$, $R_l = 0.1$, $\delta = 0.2$).

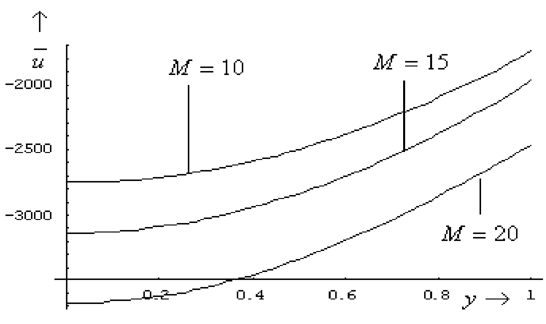

Figure 3: Effect of M on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 4$, $E_3 = 0.06$, $\mu_1 = 0.05$, $R_l = 0.1$, $\delta = 0.2$).

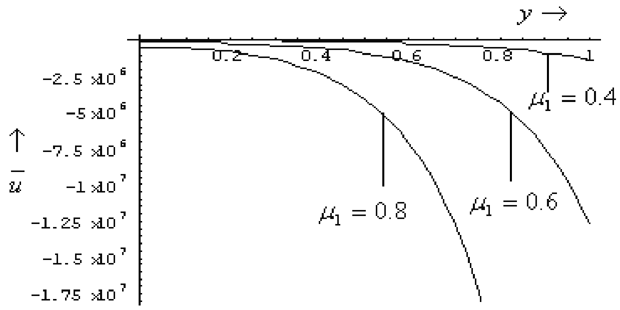

Figure 4: Effect of μ_1 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 1.0$, $E_2 = 4$, $E_3 = 0.04$, $M = 10$, $R_l = 0.1$, $\delta = 0.2$).

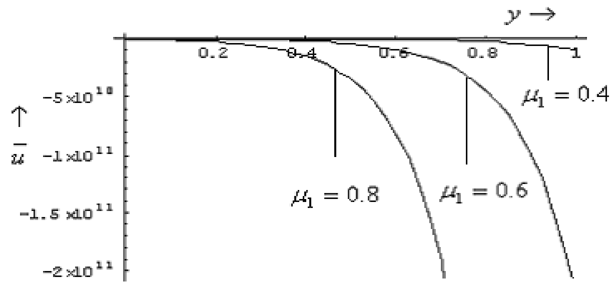

Figure 5: Effect of μ_1 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 1.0$, $E_2 = 4$, $E_3 = 0.04$, $M = 15$, $R_l = 0.1$, $\delta = 0.2$).

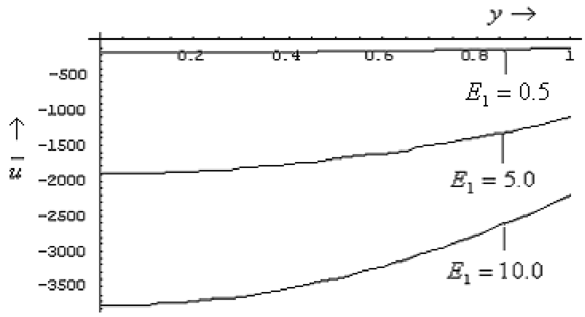

Figure 6: Effect of E_1 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_2 = 0.0$, $E_3 = 0.0$, $\mu_1 = 0.01$, $M = 15.0$, $R_l = 0.1$, $\delta = 0.2$).

From Eq. (2.6), we may note that E_1, E_2 and E_3 cannot be taken as zero simultaneously.

From the definition of M, it can be seen that as M increases, the viscous effects are dominant than the couple stress effects and hence $M \rightarrow \infty$ indicates the case of pure viscous fluid effect. Also, it can be seen from Figs. 2 and 3 that as M increases, i.e., couple stress effects decrease, the time average velocity $\bar{u}$ increases numerically.

Figs. 4 and 5 show that as cross viscosity parameter, i.e., viscosity coefficients ratio μ_1, increases, the time average velocity $\bar{u}$ increases numerically. However, this increase is predominant near walls of the channel but is insignificant in the middle of

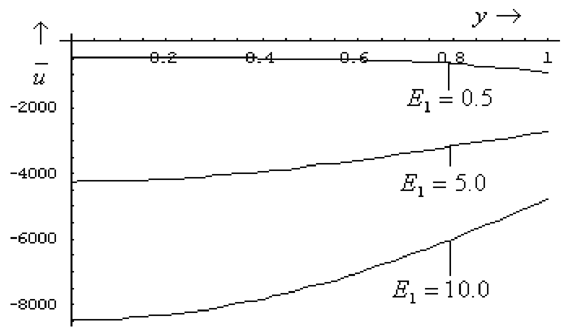

Figure 7: Effect of E_1 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_2 = 0.0$, $E_3 = 0.0$, $\mu_1 = 0.1$, $M = 15.0$, $R_l = 0.1$, $\delta = 0.2$).

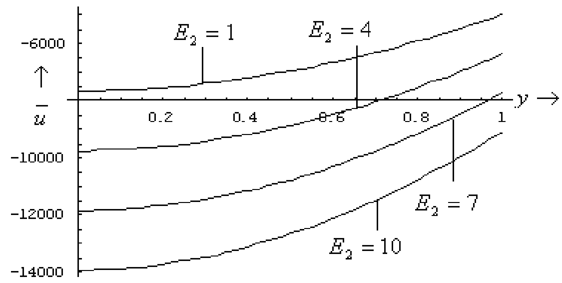

Figure 8: Effect of E_2 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 10$, $E_3 = 0.08$, $\mu_1 = 0.01$, $M = 15.0$, $R_l = 0.1$, $\delta = 0.2$).

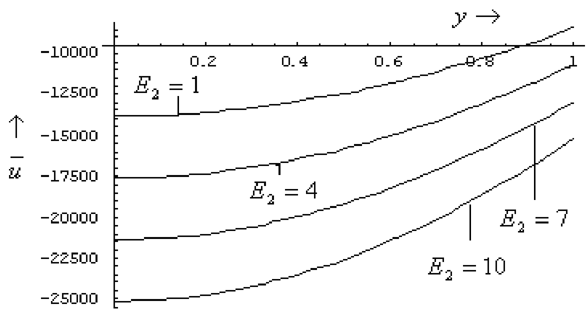

Figure 9: Effect of E_2 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 10$, $E_3 = 0.08$, $\mu_1 = 0.08$, $M = 15.0$, $R_l = 0.1$, $\delta = 0.2$).

the channel.

The effect of the rigidity parameter for the membrane (E_1) on the average velocity for the case of no stiffness in the wall ($E_2 = 0$) and perfectly elastic channel wall ($E_3 = 0$) is shown in Figs. 6 and 7. It can be noticed that the average velocity curve is parabolic in shape and $\bar{u}$ increases numerically as the rigidity of the wall increases. Further, as E_1 approaches zero, i.e., for insignificant rigidity of the wall, there is no variation in $\bar{u}$ and hence the flow is nearly uniform across the channel.

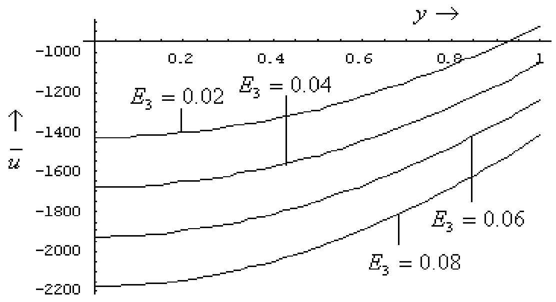

Figure 10: Effect of E_3 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 3$, $\mu_1 = 0.01$, $M = 20.0$, $R_l = 0.1$, $\delta = 0.2$).

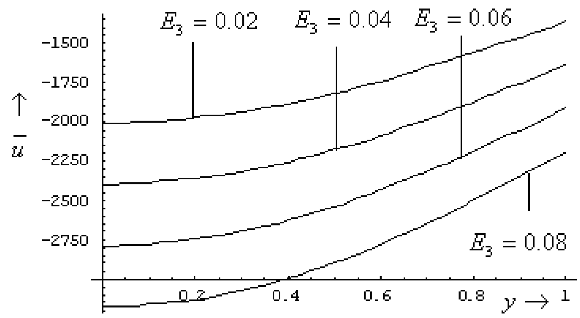

Figure 11: Effect of E_3 on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 3$, $\mu_1 = 0.05$, $M = 20.0$, $R_l = 0.1$, $\delta = 0.2$).

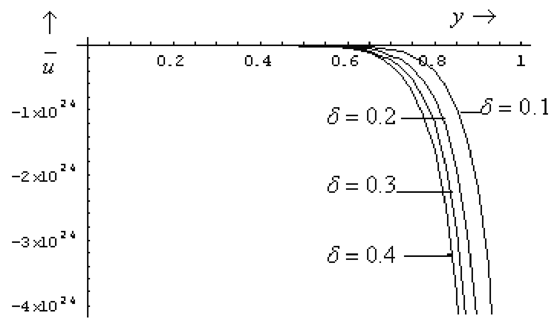

Figure 12: Effect of δ on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 3$, $E_3 = 0.06$, $\mu_1 = 5.0$, $M = 20.0$, $R_l = 0.1$).

It can be observed that (Figs. 8 and 9) the time average velocity $\bar{u}$ increases numerically with the stiffness in the wall (E_2) and also viscous damping force in the wall E_3 (Figs. 10 and 11).

The effect of wall slope parameter on the average velocity is shown in Figs. 12 and 13. $\bar{u}$ increases as wall slope increases only for higher values of y ($y > 0.6$, i.e., near the walls of the channel).

The effects of M and μ_1 on stream line pattern are shown in Figs. 14-17. It can

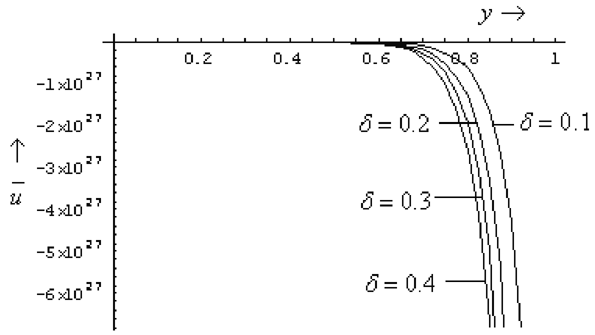

Figure 13: Effect of δ on $\bar{u}$ (Re $= 10.0$, $\varepsilon = 0.2$, $E_1 = 0.1$, $E_2 = 3$, $E_3 = 0.06$, $\mu_1 = 20.0$, $M = 20.0$, $R_l = 0.1$).

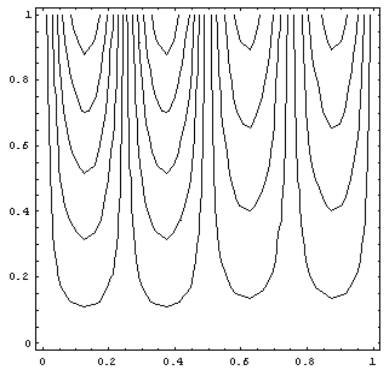

Figure 14: Effect of M on the stream line pattern of Micropolar fluid ($M = 5.0$, $\varepsilon = 0.2$, $\mu_1 = 5.0$, $E_1 = 1$, $E_2 = 4$, $E_3 = 0.02$, $R_e = 10$, $R_l = 0.1$, $\delta = 0.2$).

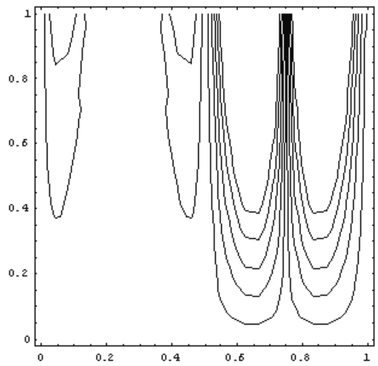

Figure 15: Effect of M on the stream line pattern of Micropolar fluid ($M = 15.0$, $\varepsilon = 0.2$, $\mu_1 = 5.0$, $E_1 = 1$, $E_2 = 4$, $E_3 = 0.02$, $R_e = 10$, $R_l = 0.1$, $\delta = 0.2$).

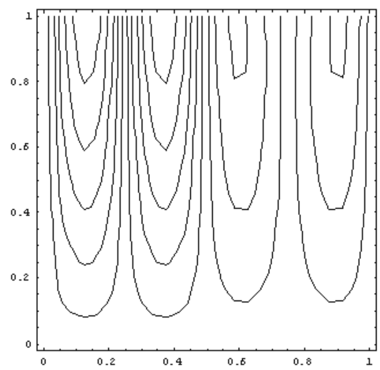

Figure 16: Effect of μ on the stream line pattern of Micropolar fluid ($M = 10.0$, $\varepsilon = 0.2$, $\mu_1 = 1.0$, $E_1 = 1$, $E_2 = 4$, $E_3 = 0.02$, $R_e = 10$, $R_l = 0.1$, $\delta = 0.2$).

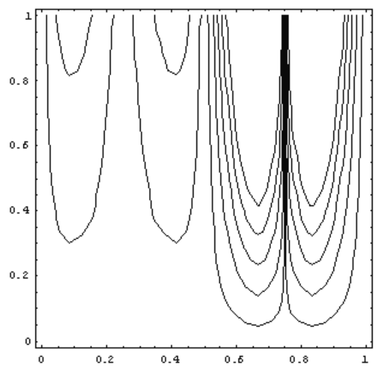

Figure 17: Effect of μ on the stream line pattern of Micropolar fluid ($M = 10.0$, $\varepsilon = 0.2$, $\mu_1 = 5.0$, $E_1 = 1$, $E_2 = 4$, $E_3 = 0.02$, $R_e = 10$, $R_l = 0.1$, $\delta = 0.2$).

be observed that the stream lines get closer, which indicates acceleration, for higher values of micropolar parameter M and cross viscosity parameter μ_1.

5 Conclusions

In this study, the peristaltic transport of an incompressible micropolar fluid in a two-dimensional channel with dynamic boundary conditions has been analyzed. The governing equations have been linearised under long wave length approximation and analytical expressions for average velocity have been derived. The effects of various parameters on time average velocity $\bar{u}$ have been studied. It is found that the time average velocity increases numerically with cross viscosity parameter, rigidity, stiffness and dissipative nature of the walls.

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