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Analysis and Numerical Approximation of an Electro-Elastic Frictional Contact Problem

El-H. Essoufi[1], El-H. Benkhira[1] and R. Fakhar[1,*]

[1] *Université Hassan I Faculté des Sciences et Téchniques, Département de Mathématiques et Informatique B.P. 577, Settat, Route de Casablanca, Settat, Maroc*

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Abstract. We consider a mathematical model which describes the static frictional contact between a piezoelectric body and a conductive foundation. A non linear electro-elastic constitutive law is used to model the piezoelectric material. The unilateral contact is modelled using the Signorini condition, nonlocal Coulomb friction law with slip dependent friction coefficient and a regularized electrical conductivity condition. Existence and uniqueness of a weak solution is established. A finite elements approximation of the problem is presented, a priori error estimates of the solutions are derived and a convergent successive iteration technique is proposed.

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Key words: Electro-elastic static problem, piezoelectric materials, unilateral contact, nonlocal Coulomb's friction, variational inequality, fixed point, finite element approximation, iterative method, error estimates.

1 Introduction

The piezoelectric effect has important uses in modern engineering because it expresses the relation between the electrical and mechanical fields. The effect known as piezoelectricity was discovered by brothers Pierre and Jacques Curie in 1880. They found out when a mechanical stress was applied on some crystals, electrical charges appeared and conversely, the production of stress or strain when an electric field is applied. The piezoelectric materials can be divided in two main groups : crystals and ceramics. The most well-known piezoelectric material is quartz SiO2, also ceramics (BaTiO3, KNbO3, LiNbO3, etc.). General models for elastic materials with piezoelectric effects can be found in [16–18, 22, 23] and, more recently, in [4, 10, 21].

Currently, there is a considerable interest in the study of contact problems involving piezoelectric materials. Thus, static frictional contact problems for electro-elastic

*Corresponding author.
Email: essoufi@gmail.com (El-H. Essoufi), benkhirahassan@yahoo.fr (El-H. Benkhira), rachid-fakhar@yahoo.fr (R. Fakhar)

materials were studied in [2,5,13,14,20], under the assumption that the foundation is insulated, and in [15] under the assumption that the foundation is electrically conductive. Exemple of quasistatic contact model in which the foundation is supposed to be conductive was investigated in [3,12].

In this paper we investigate a mathematical model which describes the static frictional contact between a piezoelectric body and a foundation. The body is supposed to be electro-elastic, with a non-linear elasticity operator. Unlike the models considered in [14,15,20], in the present paper we assume that the contact is modelled using the Signorini condition, nonlocal Coulomb friction law with slip dependent friction coefficient and a regularized electrical conductivity condition, taking into account the conductivity of the foundation as in [12], which involve a coupling between the mechanical and the electrical unknowns. This situation leads to a variational problem which is in form of a coupled system of quasi-variational inequality and non-linear variational equation. To our knowledge, this model has not been studied yet and no result has been obtained for this type problem. We establish the existence and uniqueness of weak solution to this model. Inspired from [8,11], we define the finite elements approximation of the problem and derive the error estimates on the solutions. Then, we introduce an iterative method to solve the nonlinear contact problem, which converges under certain assumptions. An important continuation of this paper consists in the numerical analysis of the model, including numerical simulations will be presented in a forthcoming work.

The paper is structured as follows. In Section 2 we present the model of equilibrium process of the elastic piezoelectric body in frictional contact with a conductive foundation. In Section 3 we introduce the functional spaces for various quantities, list the assumptions on given data and derive the weak formulation of the problem. Then, in Section 4 we state and prove our main existence and uniqueness result, Theorem 4.1. The proofs of these theorems are carried out in several steps and are based on an abstract result in the study of elliptic variational inequalities and Schauder fixed point technique. Finally, in Section 5 we study the finite element approximation of the variational formulation of problems. We prove C`ea´s type inequalities, from which we can conclude the convergence of the finite element method and derive order error estimates under appropriate regularity assumptions on the solution. We introduce an iterative method to solve the resulting finite element system, which converges under certain assumptions.

2 Problem statement

Let

$$\Omega \subset \mathbb{R}^d, \quad d = 2,3,$$

be the reference domain occupied by the electro-elastic body which is supposed to be