Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 3, pp. 389-398

DOI: 10.4208/aamm.09-m0982
June 2010

On Lateral-Torsional Buckling of Non-Local Beams

N. Challamel[1,*] and C. M. Wang[2]

[1] *Université Européenne de Bretagne, INSA de Rennes-LGCGM, 20, avenue des Buttes de Coësmes, 35043 Rennes cedex, France*

[2] *Department of Civil Engineering, National University of Singapore, Kent Ridge Crescent, Singapore 119260*

Received 25 August 2009; Accepted (in revised version) 18 March 2010
Available online 25 March 2010

Abstract. Nonlocal continuum mechanics allows one to account for the small length scale effect that becomes significant when dealing with micro- or nano-structures. This paper deals with the lateral-torsional buckling of elastic nonlocal small-scale beams. Eringen's model is chosen for the nonlocal constitutive bending-curvature relationship. The effect of prebuckling deformation is taken into consideration on the basis of the Kirchhoff-Clebsch theory. It is shown that the application of Eringen's model produces small-length scale terms in the nonlocal elastic lateral-torsional buckling moment of a hinged-hinged strip beam. Clearly, the non-local parameter has the effect of reducing the critical lateral-torsional buckling moment. This tendency is consistent with the one observed for the in-plane stability analysis, for the lateral buckling of a hinged-hinged axially loaded column. The lateral buckling solution can be derived from a physically motivated variational principle.

AMS subject classifications: 34D05, 34D20, 74B05, 74B20, 74K15, 74M25.

Key words: Lateral-torsional buckling, Kirchhoff-Clebsch theory, Eringen's model, nonlocal theory, nanostructures.

1 Introduction

Nonlocal elasticity models abandon the classical assumption of locality, and admit that stress depends not only on the strain at that point but on the strains of every point on the body. Local elasticity is inherently size-independent. In contrast, nonlocal continuum mechanics allows one to account for the small length scale effect that can become significant when dealing with small scale structures (typically nanostructures). In fact, small length scale phenomenon is linked to the atomistic structure of the lattice material. Therefore, in recent years, the nonlocal continuum mechanics has attracted the attention of many researchers who worked on the analysis of micro/nano structures.

*Corresponding author.
Email: noel.challamel@insa-rennes.fr (N. Challamel), cvewcm@nus.edu.sg (C. M. Wang)

More specifically, there has been a considerable interest in the extension of local beam theory to nonlocal beams. These articles presented simplified non-local elastic models for the bending, buckling or vibration analyses of small-scale beams/rods, e.g., [1–11]. A systematic rational procedure of non-local elasticity framework is established in Eringen's papers [12, 13]. Non-local field theory of mechanics has been applied to some various engineering problems, such as dispersion of phonon, Rayleigh wave, and stress concentration at the crack tip. The nonlocal model of Eringen [14] is chosen in this paper. This model can be presented in the one-dimensional version as

$$\sigma - l^2\sigma'' = E\varepsilon, \tag{1.1}$$

where σ is the uniaxial stress, ε is the uniaxial strain, E is the Young modulus, l is an additional length scale, specific of the nonlocal constitutive law and the double prime denotes the second order derivative of the quantity with respect to the axial coordinate of the beam. The value of l can be identified from atomistic simulations, or using the dispersive curve of the Born-Kármán model of lattice dynamics (see for instance [15, 16])

$$l \cong 0.386a, \tag{1.2}$$

where a is the distance between atoms. Zhang et al. [17] compared the critical buckling load obtained from of a nonlocal continuum mechanics approach (based on Eringen's nonlocal theory) with some Molecular Dynamics simulation. They discussed the best identification of the length scale of Eringen's nonlocal model.

The differential equation (1.1) clearly shows that the stress can be expressed as an integral of the strain variable where the weighting function is the Green's function of the differential system associated to relevant boundary conditions. The in-plane stability of nonlocal beams was first studied by Sudak [2] who used Eringen's constitutive law [14] for Euler-Bernoulli beam models. The buckling solutions have since been extended to nonlocal Timoshenko beam models (see for instance [6] or [7]). Wang et al. [18] recently investigated the postbuckling problem of cantilevered nano rods/tubes under an end concentrated load. Eringen's nonlocal beam theory is used to account for the small length scale effect.

These studies were mainly focused on straight beams, but the dynamics behaviour of nonlocal arches has been also theoretically investigated [19]. The results obtained for one-dimensional nonlocal media (nonlocal beam mechanics) have been recently extended to two-dimensional media such as nanoplates (see for instance [20] or [21]). However, the out-of-plane stability behaviour of nonlocal beams has never been investigated up to now, to the authors' knowledge. This paper aims to contribute to the understanding of lateral-torsional buckling of nonlocal beams. The behaviour of micro- or nano-cantilevers is potentially concerned by this theoretical study, since they are used as sensors in atomic force microscopy [22]. In Atomic Force Microscopy, a tip located close to the free end of the cantilever beam is utilized as a probe and can interact with a surface. Even if the specific cantilever case will not be treated in this study, the derivation of some theoretical solutions in the simple uniform bending case will