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Ma's Variation of Parameters Method for Fisher's Equations

Syed Tauseef Mohyud-Din[1,*] and Ahmet Yıldırım[2]

[1] *HITEC University Taxila Cantt Pakistan*

[2] *Ege University, Department of Mathematics, 35100 Bornova-İzmir, Turkey*

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Abstract. In this paper, we apply Ma's variation of parameters method (VPM) for solving Fisher's equations. The suggested algorithm proved to be very efficient and finds the solution without any discretization, linearization, perturbation or restrictive assumptions. Numerical results reveal the complete reliability of the proposed VPM.

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Key words: Variation of parameters method, variational iteration method, nonlinear problems, Fisher's equation, nonlinear diffusion equation, error estimates.

1 Introduction

Nonlinear partial differential equations arise in almost all areas of the natural and engineering sciences [1,2,19–21,27,28]. A particular class of equations is those modeling nonlinear reaction and diffusion phenomena [19–21,27,28]. The one-dimensional Fisher equation [19–21, 27, 28] provides an example for which the diffusion is linear and the reaction term is quadratic in the dependent variable. However, many cases occur in which the diffusion term is either nonlinear or the diffusion coefficients are functions of the dependent variable, see [19–21,27,28] and the references therein. The general properties of the solutions to such equations often depends on the given initial or boundary values selected, consequently a variety of possible behaviors may exist, including wave-like shock solutions [19–21, 27, 28]. Several numerical and analytical methods including Laurent series, tanh, finite difference and Adomian's decomposition have been developed for solving Fisher's equation, see [19–21,27,28] and the references therein. Most of these techniques have the inbuilt deficiencies. Ma et al. [3–5] used variation of parameters for solving involved non-homogeneous partial

*Corresponding author.
Email: syedtauseefs@hotmail.com (S. T. Mohyud-Din)

differential equations and obtained solution formulas helpful in constructing the existing solutions coupled with a number of other new solutions including rational solutions, solitons, positions, negatons, breathers, complextions and interaction solutions of the KdV equations. It is worth mentioning that Ma et al. [3–18] also developed and introduced some other very reliable and revolutionary new techniques also for solving physical problems of nonlinear sciences. The basic inspiration of this paper is the extension of Ma's variation of parameters method [3–5, 22–26] for solving Fisher's equations. It is to be highlighted that such equations arise very frequently in nonlinear sciences and mathematical physics, see [19–21, 27, 28] and the references therein. The VPM is applied without any discretization, perturbation, transformation or restrictive assumptions and is free from round off errors. We apply the proposed VPM for all the nonlinear terms in the problem without discretizing either by finite difference or spline techniques at the nodes, involves laborious calculations coupled with a strong possibility of the ill-conditioned resultant equations which is a too complicated to solve. Moreover, unlike the method of separation of variables that requires initial and boundary conditions, the VPM provides the solution by using the initial conditions only. It is observed that proposed VPM is also easier to implement as compare to the traditional decomposition method since it is independent of the complexities arising in calculating the so-called Adomian's polynomials. We have also applied variational iteration method for solving Fisher's equations and got results which are in full agreement with the results obtained by Ma's variation of parameters method.

2 Variation of parameters method (VPM)

Consider the following second-order partial differential equation

$$y_n = f(t, x, y, z, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}), \tag{2.1}$$

where t such that $(\infty < t < \infty)$ is time, and f is linear or non linear function of y, y_x, y_y, y_z, y_{xx}, y_{yy}, y_{zz}. The homogeneous solution of (2.1) is

$$y(t, x, y, z) = A + Bt,$$

where A and B are functions of x, y, z and t. Using Variation of parameters method, we have following system of equations

$$\frac{\partial A}{\partial t} + \frac{\partial B}{\partial t} = 0,$$
$$\frac{\partial B}{\partial t} = f,$$

and hence

$$A(x, y, z, t) = D(x, y, z) - \int_0^t sfds,$$
$$B(x, y, z, t) = C(x, y, z) - \int_0^t fds,$$

therefore,

$$y(x,y,z,t) = y(x,y,z,0) + ty_1(x,y,z,0) + \int_0^t (t-s) f(s,x,y,z,y_x,y_y,y_z,y_{xx},y_{yy},y_{zz}) ds,$$

which can be solved iteratively as [3–5,22–26],

$$y^{k+1}(x,y,z,t) = y(x,y,z,0) + ty_1(x,y,z,0) + \int_0^t (t-s) f(s,x,y,z,y^k_x,y^k_y,y^k_z,y^k_{xx},y^k_{yy},y^k_{zz}) ds,$$

where $k = 0,1,2,\cdots$.

3 Numerical applications

In this section, we apply Ma's variation of parameters method (VPM) for solving Fisher's equations. Numerical results show the complete reliability and efficiency of the proposed VPM.

Example 3.1. Consider Fisher's equation of the following form

$$u_t(x,t) - u_{xx}(x,t) - u(x,t)\big(1 - u(x,t)\big) = 0, \tag{3.1}$$

with initial conditions

$$u(x,0) = \beta. \tag{3.2}$$

Applying variation of parameters method (VPM)

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \left(\frac{\partial^2 u_n(x,\tau)}{\partial x^2} + u_n(1-u_n) \right) d\tau.$$

Following approximant are obtained

$$\begin{aligned}
&u_0(x,t) = \beta,\\
&u_1(x,t) = \beta + \beta(1-\beta)t,\\
&u_2(x,t) = \beta + \beta(1-\beta)t + \frac{t^2}{2!}\beta(1-3\beta+2\beta^2) - \frac{t^3}{3}\beta^2(-1+\beta)^2, \ \cdots .
\end{aligned}$$

The series solution is given by

$$\begin{aligned}
u(x,t) = \beta + \beta(1-\beta)t &+ \frac{t^3}{3!}\beta(1-\beta)(1-6\beta+6\beta^2) + \frac{t^4}{3}(-1+\beta)^2\beta^2(-1+2\beta)\\
&- \frac{t^5}{60}(-1+\beta)^2\beta^2(3-20\beta+20\beta^2) + \frac{t^6}{18}(-1+\beta)^3\beta^3(-1+2\beta)\\
&- \frac{t^7}{63}(-1+\beta)^4\beta^4 + \cdots ,
\end{aligned} \tag{3.3}$$

Table 1: Numerical results for Fisher's equation.

	$\beta=0.2$	$\beta=0.8$
t	*E (VPM)	*E (VPM)
0	0	0
0.2	6.19201E-06	5.57339E-06
0.4	1.03635E-04	1.13137E-04
0.6	5.45505E-04	4.00147E-04
0.8	1.78050E-03	1.18584E-03
1	4.45699E-03	1.99502E-03

*Error = Exact solution − series solution

and in a closed form by

$$u(x,t)=\frac{\beta\exp t}{1-\beta+\beta\exp t}.$$

Table 1 exhibits errors obtained by applying variation of parameters method (VPM). Now, we apply variational iteration method for solving (3.1)-(3.2). The correction functional is given by

$$u_{n+1}(x,t)=u_0(x,t)+\int_0^t\lambda(s)\left(\frac{\partial u_n}{\partial s}-\frac{\partial^2\tilde{u}_n}{\partial x^2}-\tilde{u}_n(1-\tilde{u}_n)\right)ds.$$

Making the above functional stationary, the Lagrange multiplier can be identified as $\lambda=-1$, we get

$$u_{n+1}(x,t)=u_0(x,t)-\int_0^t\left(\frac{\partial u_n}{\partial s}-\frac{\partial^2 u_n}{\partial x^2}-u_n(1-u_n)\right)ds.$$

Following approximant are obtained

$$\begin{aligned}&u_0(x,t)=\beta,\\&u_1(x,t)=\beta+\beta(1-\beta)t,\ \ \cdots.\end{aligned}$$

Consequently, we obtained a series solution which is in full agreement with (3.3).

Example 3.2. Consider the Fisher's equation of the following form

$$u_t-u_{xx}-6u(1-u)=0, \tag{3.4}$$

subject to the initial conditions

$$u(x,0)=(1+e^x)^{-2}. \tag{3.5}$$

Applying variation of parameters method (VPM)

$$u_{n+1}(x,t)=u_0(x,t)+\int_0^t\left(\frac{\partial^2 u_n(x,\tau)}{\partial x^2}+6u_n(1-u_n)\right)d\tau.$$

Following approximant are obtained

$$\begin{aligned} u_0(x,t) &= \frac{1}{(1+e^x)^2}, \\ u_1(x,t) &= \frac{1}{(1+e^x)^2} + \frac{10e^x t}{(1+e^x)^3}, \quad \cdots . \end{aligned}$$

The series solution is given by

$$\begin{aligned} u(x,t) = & \frac{25e^x}{3(1+e^x)^6}\Big(5-6e^x-15e^{2x}+20e^{3x}\Big)t^3 - \frac{50e^{2x}}{(1-e^x)^8}\Big(-17+5e^x \\ & + 52e^{2x}t^4\Big) + \frac{150e^{2x}}{(1+e^x)^9}\Big(5-47e^x+20e^{3x}\Big)t^5 + \frac{10000e^{3x}(-1+2e^x)t^6}{(1+e^x)^{10}} \\ & - \frac{240000e^{4x}t^7}{7(1+e^x)^{12}} + \frac{1}{(1+e^x)^6}\Big[25e^x(1+e^x(1+e^x)^2(-1+2e^x)t^2 \\ & - 200e^{2x}t^3 + \big(1+e^x(1+10t)\big)\Big] + \cdots , \end{aligned} \tag{3.6}$$

and in a closed form by

$$u(x,t) = \Big(1+\exp(x-5t)\Big)^{-2}.$$

Table 2 exhibits errors obtained by applying variation of parameters method (VPM). Now, we apply variational iteration method for solving (3.4)-(3.5). The correction functional is given by

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \lambda(s)\left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 \tilde{u}_n}{\partial x^2} - 6\tilde{u}_n(1-\tilde{u}_n)\right) ds.$$

Making the above functional stationary, the Lagrange multiplier can be identified as $\lambda = -1$, we get

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^t \left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 \tilde{u}_n}{\partial x^2} - 6\tilde{u}_n(1-\tilde{u}_n)\right) ds.$$

Table 2: Numerical results for Fisher's equation.

	$t=0.2$	$t=0.4$
x	*E (VPM)	*E (VPM)
0	7.22002E-03	5.75298E-02
0.2	9.89049E-03	1.6115E-01
0.4	1.09765E-02	1.39113E-01
0.6	1.04039E-02	1.51579E-01
0.8	8.50732E-03	1.43529E-01
1	5.87222E-03	1.19333E-01

*Error = Exact solution − series solution

Following approximant are obtained

$$u_0(x,t) = \frac{1}{(1+e^x)^2}, \quad \cdots .$$

Consequently, we obtained a series solution which is in full agreement with (3.6).

Example 3.3. Consider the generalized Fisher's equation

$$u_t = u_{xx} + u(1-u^6), \tag{3.7}$$

subject to the initial conditions

$$u(x,0) = \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}}. \tag{3.8}$$

Applying variation of parameters method (VPM)

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \left(\frac{\partial^2 u_n(x,\tau)}{\partial x^2} + u_n(1-u_n^6) \right) d\tau.$$

Following approximant are obtained

$$\begin{aligned} u_0(x,t) &= \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}}, \\ u_1(x,t) &= \frac{1}{(1+e^{\frac{3}{2}x})^{\frac{1}{3}}} + \frac{4+e^{3x}(4+3t)+e^{\frac{3x}{2}}(8+11t)-(1+e^{\frac{3x}{2}})^2}{4(1+e^{\frac{3x}{2}})^{\frac{7}{3}}}, \quad \cdots . \end{aligned}$$

The series solution is given by

$$\begin{aligned} u(x,t) = & \frac{1}{131072(1+e^{\frac{3x}{2}})^{\frac{49}{3}}} \Big[4096 e^{\frac{3x}{2}} (1+e^{\frac{3x}{2}})^{12} \Big(-363 + 403 e^{\frac{3x}{2}} \\ & - 9e^{3x} + 9e^{\frac{9x}{2}} \Big) t^2 - 57344 e^{3x} (1+e^{\frac{3x}{2}})^{10} (11+3e^{\frac{3x}{2}})^2 t^3 \\ & - 17920 e^{\frac{9x}{2}} (1+e^{\frac{3x}{2}})^8 (11+3e^{\frac{3x}{2}})^3 t^4 - 3584 e^{6x} (1+e^{\frac{3x}{2}})^6 (11+3e^{\frac{3x}{2}})^4 t^5 \\ & - 448 e^{\frac{15x}{2}} (1+\frac{e^3x}{2})^4 (11+3e^{\frac{3x}{2}})^5 t^6 - 448 e^{\frac{15x}{2}} (1+\frac{e^3x}{2})^4 (11+3e^{\frac{3x}{2}})^5 t^6 \\ & - 32 e^{9x} (1+e^{\frac{3x}{2}})^2 (11+3e^{\frac{3x}{2}})^6 t^7 - e^{\frac{21x}{2}} (11+3e^{\frac{3x}{2}})^7 t^8 \\ & + 32768 (1+e^{\frac{3x}{2}})^{14} \Big(4 + e^{3x}(4+3t) + e^{\frac{3x}{2}}(8+11t) \Big) \Big] + \cdots , \end{aligned} \tag{3.9}$$

and in closed form by

$$u(x,t) = \left(\frac{1}{2} \tanh \left[\frac{-3}{4} \Big(x - \frac{5}{2t} \Big) \right] + \frac{1}{2} \right)^{\frac{1}{3}}.$$

Table 3: Numerical results for the generalized Fisher's equation.

	$t=0.2$	$t=0.4$
x	*E (VPM)	*E (VPM)
0	5.24926E-02	1.21845E-01
0.2	7.79547E-02	2.17494E-01
0.4	1.10805E-01	3.4171E-01
0.6	1.51375E-01	4.94354E-01
0.8	1.99601E-01	6.74017E-01
1	2.55137E-01	8.78892E-01

*Error = Exact solution − series solution

Table 3 exhibits errors obtained by applying variation of parameters method (VPM). Now, we apply variational iteration method for solving (3.7)-(3.8). The correction functional is given by

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \lambda(s) \left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 \tilde{u}_n}{\partial x^2} - \tilde{u}_n(1-\tilde{u}_n^6) \right) ds.$$

Making the above functional stationary, the Lagrange multiplier can be identified as $\lambda = -1$, we get

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^t \left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 u_n}{\partial x^2} - u_n(1-u_n^6) \right) ds.$$

Following approximant are obtained

$$u_0(x,t) = (1+e^{\frac{3}{2}x})^{-\frac{1}{3}}, \ \cdots .$$

Consequently, we obtained a series solution which is in full agreement with (3.9).

Example 3.4. Consider the nonlinear diffusion equation of Fisher type

$$u_t = u_{xx} + u(1-u)(u-a), \qquad 0 < \alpha < 1, \tag{3.10}$$

subject to the initial conditions

$$u(x,0) = \left(1 + e^{\frac{x}{\sqrt{2}}}\right)^{-1}. \tag{3.11}$$

Applying variation of parameters method (VPM)

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \left(\frac{\partial^2 u_n(x,\tau)}{\partial x^2} + u_n(1-u_n)(u-a) \right) d\tau.$$

Proceeding as before, the series solution is given by

Table 4: Numerical results for the nonlinear diffusion equation of the Fisher type.

	$t=0.6$	$t=0.8$
x	*E (VPM)	*E (VPM)
0	7.71519E-07	4.92951E-05
0.2	7.54407E-07	4.78602E-05
0.4	7.08084E-07	4.45906E-05
0.6	6.36337E-07	3.9752E-05
0.8	5.44789E-07	3.37194E-04
1	2.02937E-06	2.69298E-05

*Error = Exact solution − series solution

$$\begin{aligned} u(x,t) = &\frac{1}{96(1+e^{x\sqrt{2}})^6}\Big[e^{\frac{x}{\sqrt{2}}}\Big(-12(1-2a)^2(-1+e^{\frac{x}{\sqrt{2}}})(-1+e^{\frac{x}{\sqrt{2}}})^3t^2 \\ &+8(1-2a)^2e^{\frac{x}{\sqrt{2}}}(1+e^{\frac{x}{\sqrt{2}}})\Big(1+a-2e^{\frac{x}{\sqrt{2}}}+ae^{\frac{x}{\sqrt{2}}}\Big)t^3 \\ &+3(-1+2a)^3e^{\sqrt{2}x}t^4+48(1+e^{\frac{x}{\sqrt{2}}})^4(2+2e^{\frac{x}{\sqrt{2}}}+t-2at)\Big)\Big]+\cdots, \end{aligned} \tag{3.12}$$

and in a closed form by

$$u(x,t) = \Big(1+\exp\frac{-\xi}{\sqrt{2}}\Big)^{-1}.$$

Table 4 exhibits errors obtained by applying variation of parameters method (VPM).

Now, we apply variational iteration method for solving (3.10)-(3.11). The correction functional is given by

$$u_{n+1}(x,t) = u_0(x,t) + \int_0^t \lambda(s)\left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 \tilde{u}_n}{\partial x^2} - \tilde{u}_n(1-\tilde{u}_n)(\tilde{u}_n - a)\right)ds.$$

Making the above functional stationary, the Lagrange multiplier can be identified as $\lambda = -1$, we get

$$u_{n+1}(x,t) = u_0(x,t) - \int_0^t \left(\frac{\partial u_n}{\partial s} - \frac{\partial^2 u_n}{\partial x^2} - u_n(1-u_n)(u_n - a)\right)ds.$$

Consequently, the obtained series solution is in full agreement with (3.12).

4 Conclusions

In this paper, we applied Ma's variation of parameters method (VPM) for finding the solution of Fisher's equation, the generalized Fisher's equation, and the nonlinear diffusion equation of the Fisher type. The method can also be extended to other nonlinear evolution equations. The results clearly indicate that the proposed method is equally good for small and large time t. The method is applied in a direct way without using linearization, transformation, perturbation, discretization or restrictive assumptions. It may be concluded that VPM is very powerful and efficient in finding the analytical solutions for a wide class of boundary value problems. The method gives more realistic series solutions that converge very rapidly in physical problems. The fact that the VPM solves nonlinear problems without using the Adomian's polynomials is a clear advantage of this technique over the decomposition method.

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