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Lattice Boltzmann Modeling of Viscous Elementary Flows

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Abstract. A lattice Boltzmann method is developed for modeling viscous elementary flows. An adjustable source term is added to the lattice Boltzmann equation, which can be tuned to model different elementary flow features like a doublet or a point source of any strength, including a negative source (sink). The added source term is dimensionally consistent with the lattice Boltzmann equation. The proposed model has many practical applications, as it can be used in the framework of the potential flow theory of viscous and viscoelastic fluids. The model can be easily extended to the three dimensional case. The model is verified by comparing its results with the analytical solution for some benchmark problems. The results are in good agreement with the analytical solution of the potential flow theory.

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Key words: The Lattice Boltzmann method, Chapman-Enskog expansion, source term, point source, point sink, doublet, viscous elementary flows, potential flow theory, symmetry.

1 Introduction

The development of the lattice Boltzmann method (LBM) has rapidly grown in the last two decades. The reason for this; is the easiness of use of the technique in addition to its ability to solve a variety of problems that are impractical or even impossible to be solved by traditional computational fluid dynamics (CFD) techniques.

Since its appearance, many researchers have tried to make the method more powerful by modifying its equations to incorporate more physical phenomena. Examples of this include the lattice Boltzmann method for axisymmetric fluid flow [3], axisymmetric thermal flows [22], multiphase flow [14] and viscous fingering phenomenon [1].

The potential flow theory has always been thought of as a theory for irrational inviscid flow problems. However, a lot of recent literature investigated the potential

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use of the potential flow theory for viscous and viscoelastic fluids at low Reynolds numbers [5, 17–19].

Thus, modeling of the elementary flow features using the lattice Boltzmann method adds new capabilities to the technique and opens new frontiers for its potential use.

Several researchers have tried to solve fluid flow problems involving different types of singularities. In the case of geometrical singularities, [9] has introduced a series solution for the steady flow of a viscous fluid in the neighbourhood of a sharp corner. [4] have proposed a method to incorporate this series solution into a finite difference scheme for the solution of the stream function/vorticity formulation of the Navier-Stokes equations in the case of a rectangular re-entrant corner. The scheme is successful in overcoming the singularity problem of the velocity gradient near a sharp corner. They have used their proposed scheme to solve the flow field in a channel with a sudden contraction.

For cases where stress singularities arise from abrupt changes in boundary conditions in viscous flow problem, [2] proposed a singular finite element scheme for stokes flow and used it to solve for the flow field of the stick-slip problem. They have used special elements surrounding the singular point without a pressure node at the singular point. They showed that ordinary finite element schemes are less accurate in the neighbourhood of the singular point. The scheme is also useful for high Reynolds number flows and for Non-Newtonian fluids provided that the stresses are integrable.

The reference [7] proposed a moving mesh finite element algorithm for problems in two and three space dimensions. Their scheme is successful in solving various problems (including fluid dynamics problems) with regions of high gradients.

In this paper, an easy-to-implement and efficient way of modeling elementary flows through the incorporation of a point source, sink and a doublet using the lattice Boltzmann method is proposed for the first time. The results of the numerical simulations are compared to the analytical solution of the potential flow theory and are in excellent agreement.

Modeling elementary flow features has always been a challenge for numerical modeling. The reason is the associated mathematical singularity at the point of interest. To the best of the authors' knowledge, numerical methods based on the solution of the continuum conservation equations like finite difference and finite element have never been used to model a point source, sink or doublet using a single point in the computational domain.

Due to the continuum approach considerations, only one value for each variable can be assigned at a certain point at a specific time instant. For example, at any point only one value for the macroscopic flow velocity can be specified or calculated. However, due to the nature of the lattice Boltzmann method formulation and the fact that it does not deal directly with the macroscopic variables, the local distribution function can be formulated in such a way to allow for different proportions of particles to move along different directions, see Fig. 1. Then, these microscopic velocities stream to neighbour lattices' nodes during the streaming step. This allows for the modeling of a point source through a single lattice node.