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Comparison of Different Formulations for the Numerical Calculation of Unsteady Incompressible Viscoelastic Fluid Flow

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Abstract. In this paper we compare different methods currently used in the stabilization of numerical simulations of time-dependent viscoelastic fluid flows described with the Oldroyd-B and related models. The methods under consideration, based on the separation of newtonian-like components from the stress tensor, are applied to a finite volume analysis of two simple benchmark problems (the plane Poiseuille startup and pulsated flows), for which analytical solutions are known. The relative performances of each method are evaluated regarding stability, accuracy and efficiency.

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1 Introduction

The absence of explicit diffusive terms in the governing equations of the Oldroyd-B and related models makes the convergence of numerical iterative simulations of time dependent viscoelastic flows based on the original formulation of such models difficult if at all possible. To remedy this situation it is common practice to include diffusive terms in the equations, either by separating purely viscous components from the stress tensor (like in elastic-viscous stress splitting methods [1–3] or in solvent-polymer decompositions [4,5]) or by explicitly adding such a term and a corresponding correction in ”source” terms [6–8]. These techniques have been (and still are)

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widely applied to stabilize the numerical computation of non-newtonian flows using viscoelastic constitutive models but, to our knowledge, their relative merits have never been systematically studied. Such is the purpose of this work.

In this introduction to the problem, we first present the governing differential equations we will be dealing with, then we provide a brief description of alternative formulations currently used in numerical simulations and finally show how these formulations can be written under a common general form. We then (Section 2) present the benchmark problems used in the comparison and their analytical solutions and, in Section 3, the numerical method is briefly outlined. In Section 4 we present and discuss our results, and conclusions are drawn in the last section.

1.1 Basic equations and viscoelastic model

Fluid motion at a macroscopic level is well described by Newton's Second Law, which, per unit volume and in the absence of external forces, is written as

$$\rho\frac{d\vec{v}}{dt} = -\vec{\partial}p + \vec{\partial}\cdot\boldsymbol{\tau}, \tag{1.1}$$

for a fluid with mass density ρ moving with velocity $\vec{v}$, under a pressure field p. The stress tensor $\boldsymbol{\tau}$ obeys a constitutive equation that describes the particular stress-strain behavior of the fluid under consideration. In this work we consider the Oldroyd-B model [9], defined by the following constitutive relation

$$\boldsymbol{\tau} + \lambda_1\overset{\triangledown}{\boldsymbol{\tau}} = 2\eta_0\Big(\boldsymbol{D} + \lambda_2\overset{\triangledown}{\boldsymbol{D}}\Big). \tag{1.2}$$

Here, η_0 is the viscosity and λ_1 and λ_2 are two model parameters respectively named *relaxation time* and *retardation time*. The rate of deformation tensor $\boldsymbol{D}$ is given by

$$D_{ij} = \frac{1}{2}(\partial_i v_j + \partial_j v_i), \tag{1.3}$$

and the *convected derivative* [4] is defined in general as

$$\overset{\triangledown}{\boldsymbol{\Omega}} = \frac{d\boldsymbol{\Omega}}{dt} - \boldsymbol{\Omega}\cdot(\boldsymbol{\partial v}) - (\boldsymbol{\partial v})^T\cdot\boldsymbol{\Omega}, \tag{1.4}$$

or, more explicitly (sum over repeated index k implied),

$$\overset{\triangledown}{\Omega}_{ij} = \frac{d\Omega_{ij}}{dt} - \Omega_{ik}\partial_k v_j - \partial_k v_i\Omega_{kj}. \tag{1.5}$$

The description of a viscoelastic flow requires the simultaneous solution of Eqs. (1.1) and (1.2), together with an equation for mass conservation which, for the case here assumed of incompressible flow, reduces to a zero velocity divergence constraint ($\vec{\partial}\cdot\vec{v} = 0$), and in most practical situations numerical methods are mandatory. In a straight