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# Lattice Boltzmann and Finite Volume Simulation of Inviscid Compressible Flows with Curved Boundary

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**Abstract.** A 2D lattice Boltzmann model for inviscid compressible flows was proposed in this paper. Finite volume method was implemented on 2D curvilinear structural grids to solve the lattice BGK-Boltzmann equations. MUSCL scheme was used to perform interpolation. The obtained results agree excellently well with experimental and previous numerical results.

**AMS subject classifications**: 76P05

**Key words**: Lattice Boltzmann method, compressible flow, finite volume method, structural grids.

## 1 Introduction

As a promising method, the lattice Boltzmann method (LBM) has been widely studied in the last decades. Although LBM was derived from the lattice gas automata (LGA), it can also be derived from the continuous Boltzmann equation and it is a special finite difference form of the discrete velocity Boltzmann equation (DVBE). DVBE can be solved with finite difference method, finite volume method or finite element method. The solution of DVBE by the FVM was first conducted by Nannelli and Succi [1,2].

FVM is a method based on the weak solution of PDE and it has the feature of keeping conservation laws of physics. Due to this feature, it is a primary approach in simulations of compressible flows with discontinuities, such as shock waves and contact discontinuities. Many FVM schemes were proposed and widely used to capture discontinuities, such as TVD, MUSCL, ENO/WENO. So it is natural to use such FVM schemes to solve DVBE to simulate compressible flows with discontinuities.

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However, in the history of LBM, its applications were limited to incompressible flows. This is because LB models were derived based on low Mach number expansion of Maxwellian function [3–6]. Although some compressible LB models were proposed [7–10], the Mach number range of them is small and no supersonic numerical results were published. At the same time, some of them have many free parameters. To overcome these difficulties, we proposed a 2D model for inviscid compressible flows, D2Q13L2 (L2 means two energy-levels) in [12] from a circular function. Based on this model, we solved DVBE with the 2nd order TVD FVM on uniform rectangular grids to simulate several cases of compressible flows [11,12]. Our simulations have shown that it can simulate supersonic flows with high Mach number and strong shock waves. Based on our research, Li et al. [13] and Wang et al. [14] proposed their double ditribution function LB models for viscous compressiblel flows.

In this work, we extended the application of the D2Q13L2 model to cuvilinear structural grids to simulate flow fields with irregular boundaries. The MUSCL scheme with the smooth limiter was used to capture discontinuities. The implementation of boundary conditions for compressible flows was discussed. In order to validate the method, we simulated several test problems and compared the results with experimental data or previous results. Excellent agreement was obtained. The rest of this paper was organized as follows. Section 2 will introduce our D2Q13L2 model. Section 3 will present the FVM discretization of DVBE. Section 4 is about implementations of boundary conditions. Numerical applications will be presented in Section 5. Finally, Section 6 concludes the present work.

## 2 D2Q13L2 model for 2D inviscid compressible flows

The detailed derivation of the D2Q13L2 model was presented in [11,12]. Here, we only briefly describe it. First, our derivation is not based on Maxwellian function which is complicate and difficult to mathematically mannipulate. Alternatively, we proposed a simplified function. For 2D problems, the form of the simplified function is

$$g = \begin{cases} \dfrac{\rho}{2\pi c}, & \text{if } \|\boldsymbol{\xi} - \boldsymbol{u}\| = c \equiv \sqrt{D(\gamma-1)e} \\ & \quad \text{and } \lambda = e_p = \Big[1 - \dfrac{D}{2}(\gamma-1)\Big]e, \\ 0, & \text{else}, \end{cases} \tag{2.1}$$

where $\boldsymbol{\xi}$ is particle velocity, $\lambda$ is rest energy of particles, $c$ is an effective peculiar velocity, $\boldsymbol{u}$ is the mean flow velocity, $e$ is the mean internal energy, $\gamma$ is the specific heat ratio of the gas, $D$ is the space dimension. This function means that all mass, specific momentum and energy concentrate on a circle located in a 3D space of $\xi_x$-$\xi_y$-$\lambda$ (as shown in Fig. 1). Although this function is very simple, it satisfies all needed statistical relations to make BGK DVBE recover the compressible NS equations.

Second, based on this circular function, we discretized it onto some fixed points in the 3D space of $\xi_x$-$\xi_y$-$\lambda$ to construct a LB model. We adopted the Lagrangian interpolation to assign the circular function onto a set of discrete points without small Mach