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Semiclassical Axisymmetric Lattice Boltzmann Method

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Abstract. A semiclassical lattice Boltzmann method is presented for axisymmetric flows of gas of particles of arbitrary statistics. The method is first derived by directly projecting the Uehling-Uhlenbeck Boltzmann-BGK equations in two-dimensional rectangular coordinates onto the tensor Hermite polynomials using moment expansion method and then the forcing strategy of Halliday et al. (Phys. Rev. E., 64 (2001), 011208) is adopted and forcing term is added into the resulting microdynamic evolution equation. The determination of the forcing terms is dictated by yielding the emergent macroscopic equations toward a particular target form. The correct macroscopic equations of the incompressible axisymmetric viscous flows are recovered through the Chapman-Enskog expansion. Computations of uniform flow over a sphere to verify the method are included. The results also indicate distinct characteristics of the effects of quantum statistics.

AMS subject classifications: 76P05, 82B40

Key words: Semiclassical lattice Boltzmann method, axisymmetric flows, flow over a sphere, Bose gas, Fermi gas.

1 Introduction

In the past two decades, significant advances have been accomplished in the development of the lattice Boltzmann methods [1–4] based on classical Boltzmann equations with the relaxation time approximation of Bhatnagar, Gross and Krook (BGK) [5]. The lattice Boltzmann method (LBM) has illustrated its capability for simulating hydrodynamic systems, magnetohydrodynamic systems, multi-phase and multi-component fluids, multi-component flow through porous media and complex fluid systems, see [6]. The lattice Boltzmann equations (LBE) can also be directly derived in *a priori* manner from the continuous Boltzmann-BGK equation [7, 8]. Most of the classical LBMs are accurate up to the second order, i.e., Navier-Stokes hydrodynamics and have

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not been extended beyond the level of the Navier-Stokes hydrodynamics. A systematical method [9, 10] was proposed for kinetic representation of hydrodynamics beyond the Navier-Stokes equations using Grad's moment expansion method [11].

It is also observed that most of the existing lattice Boltzmann methods are focused on hydrodynamics of classical particles. However, modern development in nanoscale transport requires carriers of particles of arbitrary statistics [12]. The extension and generalization of the successful classical LBM to treat particles of arbitrary statistics is thus desirable. Analogous to the classical Boltzmann equation, a semiclassical Boltzmann equation which taking into account the effect of quantum statistics has been developed by Uehling and Uhlenbeck (UUB) [13]. To circumvent the mathematical difficulty of the the collision term, BGK-type relaxation time models to capture the essential properties of carrier scattering mechanisms can be similarly devised for the Uehling-Uhlenbeck Boltzmann equation for various carriers and have been widely used in carrier transport [14]. Recently, a semiclassical gas-kinetic scheme [15] has been developed for the hydrodynamic transport based on the Uehling-Uhlenbeck Boltzmann-BGK (UUB-BGK) equation. Also, a two-dimensional semiclassical lattice Boltzmann method for the UUB-BGK equation based on D2Q9 lattice model [2] and Grad's moment expansion method has been presented [16]. Hydrodynamics based on moments up to second and third order expansions are presented. Simulations of flow over a circular cylinder at low Reynolds numbers have been tested and have been found in good agreement with previous available results.

One of the most common and important classes of fluid dynamical problems is the axisymmetric flow in which flow symmetry with respect to an axis can be identified. Classical axisymmetric lattice Boltzmann method was first proposed by Halliday et al. [17] using a forcing strategy. By introducing source terms, the macroscopic equations for the axisymmetric flows can be recovered. The method of Halliday et al. has been successfully applied to a number of axisymmetric flow problems [18–26]. Recently, an interesting lattice Boltzmann model for axisymmetric flows based on Boltzmann-BGK equation in cylindrical coordinates has been proposed [27].

The main objective of this work is to present the semiclassical axisymmetric lattice Boltzmann method for axisymmetric flow of gases of arbitrary statistics. The method of Halliday et al. [17] is adopted and forcing terms are added into the two-dimensional semiclassical Boltzmann-BGK equation which are consistent in dimension with the lattice Boltzmann equation. The forcing terms are determined by demanding the emergent macroscopic equations toward a particular target form. The set of correct macroscopic equations for incompressible axisymmetric flows can be recovered through the Chapman-Enskog multiscale analysis of the semiclassical LBM.

This paper is organized as follows. Section 2 gives a brief description of element of semiclassical kinetic theory. The basic two-dimensional semiclassical lattice Uehling-Uhlenbeck Boltzmann-BGK method is described in Section 3. The derivation of the axisymmetric semiclassical LBM is given in Section 4. Simulations of uniform flow over a sphere using the present method are given in Section 5. Concluding remarks are given in Section 6.