Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **2**, No. 5, pp. 677-684

DOI: 10.4208/aamm.10-10S10
October 2010

# Lattice Boltzmann Method for Thermocapillary Flows

Lin Zheng[1], Zhaoli Guo[1,*], Baochang Shi[1] and Chuguang Zheng[1]

[1] *National Laboratory of Coal Combustion, Huazhong University of Science and Technology, Wuhan 430074, China*

Received 5 March 2010; Accepted (in revised version) 3 April 2010
Available online 13 July 2010

**Abstract.** In this paper, we apply a recently proposed thermal axisymmetric lattice Boltzmann model to the thermocapillary driven flow in a cylindrical container. The temperature profiles and isothermal lines at the free surface with Prandtl (Pr) number fixed at 0.01 and Marangoni (Ma) number varying from 10 to 500 are measured and compared with the previous numerical results. In addition, we also give the numerical results for different Ma numbers at Pr=1.0. It is shown that present results greed well with those reported in previous studies.

**AMS subject classifications**: 65C20, 80A20, 76R10
**Key words**: Lattice Boltzmann method, axisymmetric flows, thermocapillary flows.

## 1 Introduction

Surface tension gradient at a free surface could induce a viscous driving flow [1–3]. This phenomena (usually called thermocapillary convection) is often encountered in many industrial processes. The subject of thermocapillary convection has been an interesting area for the science and engineering due to its complex flow filed and practical applications such as crystal growth melts and the convective flows in the microgravity environment.

In some special cases, e.g., thermocapillary convection in an axisymmetric configuration, such flows can be regarded as a quasi-two-dimensional problems. Many traditional methods such as finite difference method, finite volume method, vorticity-stream method, SIMPLE method have been applied to this field. It should be mentioned that, in the last two decades, lattice Boltzmann equation (LBE) has been rapidly developed as an effective and promising numerical algorithm for computational fluid dynamics [4–6], which has also been applied to axisymmetric flows [7–12].

Thermocapillary flow induced by the temperature gradient in the rectangular cavity has been widely studied by traditional methods and LBE. However, to the authors'

*Corresponding author.
*Email:* zlguo@mail.hust.edu.cn (Z. L. Guo)

 

acknowledge, there are many attempts to apply the traditional methods to the thermocapillary flow in an axisymmetric cylindrical cavity, but it's quite rare for LBE. Therefore, in present paper, we will apply a recent thermal axisymmetric model [11] to the thermocapillary driven flow in a cylindrical container by a motionless surface with constant wall temperature and straight, undeformable lateral free surface boundary with a steady heat flux. Numerical simulations have been conducted at different Pr and Ma numbers and the numerical results indicate that present results agree well with other existing work [1].

The outline of the paper is as follows: in Section 2 we give a brief description of the physical problem. In Section 3 the axisymmetric thermal LBE model is introduced. Then we demonstrate some numerical simulations to validate the results in Section 4 and the conclusions are drawn in Section 5.

## 2 Physical problem description

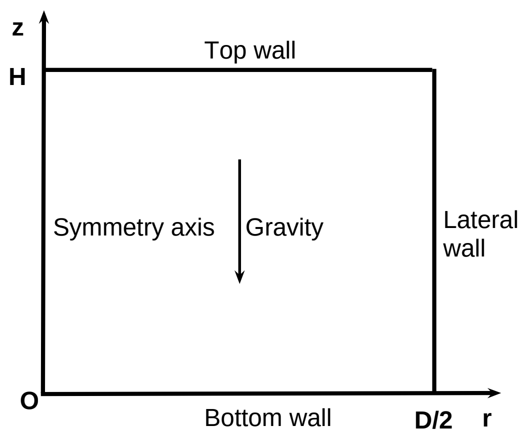


Figure 1: Sketch of the cylinder flow.

The physical configuration in Fig. 1 is axisymmetric, limited by motionless surface with constant wall temperature. The lateral boundary is the free surface which is taken to be straight and undeformable. The ratio of the radius and the height is fixed at 1/2, the gravity force and the azimuthal velocity is ignored in this case. Under these conditions, the liquid motion and temperature distribution for this problem are governed by the following dimensionless equations

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r}+\frac{\partial u_z}{\partial z}=0, \tag{2.1a}$$

$$\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+u_z\frac{\partial u_r}{\partial z}=-\frac{\partial p}{\partial r}+\mathrm{Pr}(\nabla^2 u_r-\frac{u_r}{r^2}), \tag{2.1b}$$