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Modelling and Analysis of a Class of Metal–Forming Problems

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Abstract. A class of steady-state metal-forming problems, with rigid-plastic, incompressible, strain-rate dependent material model and nonlocal Coulomb's friction, is considered. Primal, mixed and penalty variational formulations, containing variational inequalities with nonlinear and nondifferentiable terms, are derived and studied. Existence, uniqueness and convergence results are obtained and shortly presented. A priori finite element error estimates are derived and an algorithm, combining the finite element and secant-modulus methods, is utilized to solve an illustrative extrusion problem.

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Key words: Steady-state metal-forming, rigid-plastic material, nonlocal friction, variational formulations, weak solutions, secant-modulus method, FE analysis.

1 Introduction

The computational and experimental study of metal-forming processes has shown that the flow theory of plasticity [1–3] adequately approximates the material behaviour for most of them [4–7], as the frictional contact conditions also significantly influence the results. Due to similarity with the the contact problems in elasticity [8–15], corresponding metal-forming, or plastic flow contact problems, could be formulated and mathematically analysed. This direction of analysis has been followed for example in [16–20] and references therein, where steady-state wire-drawing, extrusion and rolling problems, with linear rigid-viscoplastic Bingham material model [3, 8–10], or nonlinear rigid-viscoplastic material models [4–7] and normal compliance, or nonlocal contact and Coulomb's friction models [11–15], have been formulated and studied. Variational inequality formulations have been derived and existence and uniqueness

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results have been obtained. The solution of the resulting nonlinear variational problems, requires appropriate succesive linearization methods [9, 11, 12, 21], finite element methods and computational algorithms [11,21,22], to be applied. In [17–20] for example, the secant-modulus method, proposed by Kachanov [9] for solving nonlinear variational problems in the deformation theory of plasticity, has been extended to nonlinear variational inequalities, as in [20] a finite element–secant-modulus computational algorithm is proposed and used.

In this work, a class of metal-forming problems is considered, describing steady-state drawing and extrusion, with nonlocal Coulomb's friction through a rigid die, of an isotropic, rigid-plastic, strain-rate sensitive incompressible metallic strip (workpiece). Primal, mixed and penalty variational inequality formulations, with strongly nonlinear and nondifferentiable terms, are derived and studied. Under restrictions on the material characteristics, existence, uniqueness and convergence results are obtained and shortly presented. Finite element approximations are performed, a priori error estimates are derived and an algorithm, combining the finite element and the secant-modulus method, is utilized to solve an illustrative extrusion problem.

2 Statement of the problem

We suppose that a metallic workpiece occupies the domain $\Omega\subset \mathbb{R}^k$ (k=2, 3), with sufficiently regular boundary Γ, constituting of six open, disjoint subsets (Fig. 1). By Γ_1 and Γ_5 the vertical rear and front ends of the workpiece are denoted. A constant process velocity is prescribed on Γ_1 at extrusion, as Γ_5 is assumed free of tractions, or on Γ_5 at drawing, as then Γ_1 is assumed tractions free. The boundary $\Gamma_2 \cup \Gamma_4$ is also assumed tractions free. The contact boundary is denoted by Γ_3. Due to the symmetry, only one half of the workpiece is considered, as by Γ_6 the boundary of symmetry is denoted. We shall further identify the points of $\bar{\Omega}=\Omega \cup \Gamma$ by their cartesian coordinates $\mathbf{x}=\{x_i\}$ and shall use the standard indicial notation and summation convention. Let us denote by

$$\mathbf{u}(\mathbf{x}) = \{u_i(\mathbf{x})\}, \quad \boldsymbol{\sigma}(\mathbf{x}) = \{\sigma_{ij}(\mathbf{x})\}, \quad \dot{\boldsymbol{\varepsilon}}(\mathbf{x}) = \{\dot{\varepsilon}_{ij}(\mathbf{x})\}, \quad (1 \le i, j \le k),$$

the velocity vector, stress and strain-rate tensors respectively and by

$$\bar{\sigma} = \sqrt{\frac{3}{2} s_{ij} s_{ij}}, \qquad \dot{\bar{\varepsilon}} = \sqrt{\frac{2}{3} \dot{e}_{ij} \dot{e}_{ij}}, \tag{2.1}$$

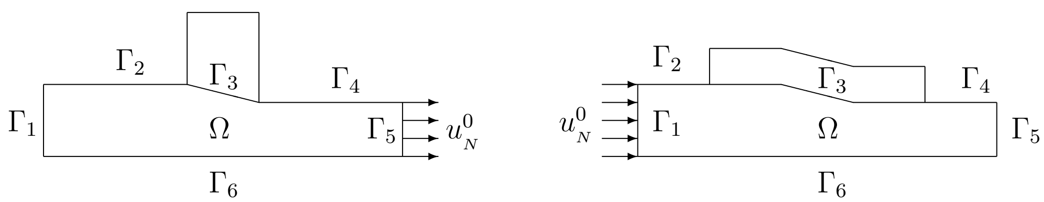

Figure 1: Strip drawing and extrusion problems.