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Total Reflection and Cloaking by Triangular Defects Embedded in Zero Index Metamaterials

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Abstract. In this work, we investigate wave propagation through a zero index metamaterial (ZIM) waveguide embedded with triangular dielectric defects. We provide a theoretical guidance on how to achieve total reflection and total transmission (i.e., cloaking) by adjusting the defect sizes and/or permittivities of the defects. Our work provides a systematical way in manipulating wave propagation through ZIM in addition to the widely studied dielectric defects with cylindrical and rectangular geometries.

AMS subject classifications: 78M10, 65N30, 35L15

Key words: Metamaterials, invisibility cloaks, Maxwell's equatons.

1 Introduction

The past decade saw a growing interest in the study of zero index metamaterials (ZIMs), whose permittivity and permeability are simultaneously or individually near zero. Ziolkowski [1] showed that a matched ZIM slab can be used to transform curved wave fronts into planar ones. Research shows that ZIMs can be used to squeeze electromagnetic (EM) waves [2–4], to perfectly bend and transmit EM waves [5], and to block wave transmission (resulting in total reflection) and conceal objects (cloaking) [6–11]. Such total reflection and total transmission (i.e., cloaking) are realized by embedding proper defects in ZIMs. However, almost all publications on dielectric defects are restricted to cylinderical geometry [8–10].

In this paper, we study the triangular defects case. Three most popular types of triangles are discussed. A systematic guidance for achieving total reflection and cloaking is provided. Extensive full-wave simulations are conducted to verify our analysis. We

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like to remark that cloaking realized by embedding proper defects in ZIMs is one of many strategies discussed intensively for cloaking with metamaerials in the past decade (e.g., [15–27] and references cited therein).

2 Realization of total reflection and cloaking

Consider an EM wave incident from the left into a waveguide structure illustrated in Fig. 1. The ZIM region is a metamaterial, whose relative permittivity and permeability are described by the so-called Drude model [6,12]:

$$\epsilon_1 = \mu_1 = 1 - \frac{\omega_p^2}{\omega(\omega + i\Gamma)},$$

where ω is the excitation angular frequency, ω_p denotes the plasma frequency, and Γ denotes the loss parameter. On both sides of the ZIM region are the free space regions 0 and 3. The defects in region 2, which is embedded in ZIM, are composed of $N \geq 1$ triangles with permittivity and permeability of $\epsilon_{2,k}$ and $\mu_{2,k}$, $k = 1,2,\cdots,N$, respectively. Assuming $\exp(-i\omega t)$ time harmonic factor, the EM wave in each region satisfies the Maxwell's equations: For any $m = 0,1,2,3$,

$$\boldsymbol{H}_m = \frac{1}{i\omega\mu_0\mu_m}\nabla\times\boldsymbol{E}_m, \quad \boldsymbol{E}_m = \frac{i}{\omega\epsilon_0\epsilon_m}\nabla\times\boldsymbol{H}_m. \tag{2.1}$$

Eq. (2.1) can be reduced to the Helmholtz equation:

$$(\partial_{xx} + \partial_{yy} + k_0^2\epsilon_m\mu_m)\boldsymbol{U}_m = 0, \tag{2.2}$$

where $\boldsymbol{U}_m = \boldsymbol{E}_m$ or $\boldsymbol{H}_m$, and $k_0 = \omega\sqrt{\epsilon_0\mu_0}$ is the wave vector in vacuum.

Assume that the walls of the waveguide are made of perfect electric conductor, and the incident wave ($\boldsymbol{H}$ field) propagates to the right along the x direction. By applying Faraday-Maxwell law and Stokes' theorem, we can show that the transmission coefficient

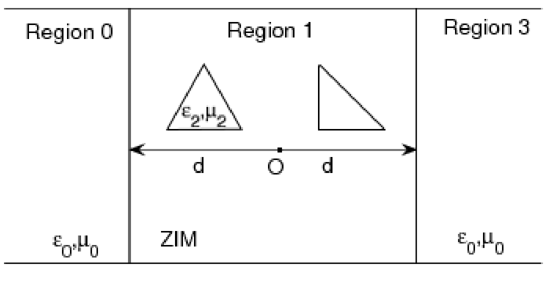

Figure 1: The schematic description of the waveguide structure with vacuum (regions 0 and 3), ZIM (region 1), and embedded triangular defects.