Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 2, pp. 187-212

DOI: 10.4208/aamm.2014.m706
April 2016

A Numerical Method on Eulerian Grids for Two-Phase Compressible Flow

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Received 29 July 2014; Accepted (in revised version) 20 January 2015

Abstract. We develop a numerical method to simulate a two-phase compressible flow with sharp phase interface on Eulerian grids. The scheme makes use of a levelset to depict the phase interface numerically. The overall scheme is basically a finite volume scheme. By approximately solving a two-phase Riemann problem on the phase interface, the normal phase interface velocity and the pressure are obtained, which is used to update the phase interface and calculate the numerical flux between the flows of two different phases. We adopt an aggregation algorithm to build cell patches around the phase interface to remove the numerical instability due to the breakdown of the CFL constraint by the cell fragments given by the phase interface depicted using the levelset function. The proposed scheme can handle problems with tangential sliping on the phase interface, topological change of the phase interface and extreme contrast in material parameters in a natural way. Though the perfect conservation of the mass, momentum and energy in global is not achieved, it can be quantitatively identified in what extent the global conservation is spoiled. Some numerical examples are presented to validate the numerical method developed.

AMS subject classifications: 65M10, 78A48

Key words: Two-phase flow, levelset, Riemann problem.

1 Introduction

Many problems in nature and engineering involve multiphase flows where the flows in different phases are depicted by an immiscible model. Numerical methods to accurately track/capture the interface between two fluids have been an area of research for decades.

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Tryggvason et al. [17] provided a detailed review on various methods used for direct simulation of multiphase flows. Broadly, these schemes can be classified into two categories: (a) Lagrangian and (b) Eulerian approach.

Lagrangian methods use marker-points connected to each other representing the phase interface, which is tracked by advecting the marker points. In one class of Lagrangian methods [1, 11, 18], the governing equations of the flow are solved on a fixed grid in an Eulerian frame. In another class of Lagrangian methods, the interface is represented by Lagrangian points and the flow field is also evaluated on these points, such as moving particle-methods [5], vortex in cell methods [6,12], and smoothed-particle hydrodynamics [7]. Pure Lagrangian methods are promising as they avoid enormous memory requirements for a three-dimensional mesh. Some of these methods automatically provide adaptive resolution in the high-curvature region [6] and have been applied successfully to many two-phase flow problems [4,19,20]. Although the accuracy of these method is promising, the topological change of the phase interface is not handled automatically, resulting in increased complexity of the algorithm for three-dimensional reconstruction of the interface from marker-points [17], high cost of finding nearest neighbors in the zone of influence of a Lagrangian point, true enforcement of continuity (or incompressibility) conditions, and problems associated with accurate one-sided interpolations near boundaries [6].

Eulerian approaches such as the volume-of-fluid (VOF) [8, 13] or the levelset method [9, 14, 16] are used extensively for two-phase flow computations, which are straightforward in implementation. In the simulation of incompressible flows, the levelset approach is criticized since it does not preserve the volume of the fluids. The VOF formulation, on the other hand, conserves the fluid volume but lacks in the sharpness of the interface. Several improvements to these methods involve combination of the two [15], and particle-levelsets [3] to improve the accuracy.

The method to be developed in this paper is a certain combination of the front tracking method and the levelset method. We use levelset function to represent the phase interface, which is advected by its normal velocity. To calculate the exchange of the flux between two fluids due to the interaction of the fluids on the phase interface and its displacement, we are tracking the characteristic lines of the flow particles. Interiori to the bulk of both fluids, the traditional conservative finite volume method is adopted. The evolving of the phase interface is following the standard approach of the levelset method. This makes us concentrate on the calculation of the interface flux. By studying the one dimensional two-phase Riemann problem, we derive an approximated flux contributed at the phase interface, including the terms both from the phase interface movement and the mass interaction. Using this approximated flux, we take the tracking along the characteristic line as a one dimensional problem locally. An aggregation algorithm is adopted to build cell patches around the phase interface so that the CFL constraint can be satisfied even some of the cells in the mesh are cut into smaller fragments by the phase interface. Numerical examples are presented to validate our numerical methods.

The rest of this paper is arranged as follows. In Section 2, the model of the two-