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Bell Polynomials to the Kadomtsev-Petviashivili Equation with Self-Consistent Sources

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Abstract. Bell Polynomials play an important role in the characterization of bilinear equation. Bell Polynomials are extended to construct the bilinear form, bilinear Bäcklund transformation and Lax pairs for the Kadomtsev-Petviashvili equation with self-consistent sources.

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Key words: Bell polynomials, bilinear form, bilinear Bäcklund transformation, Lax pair.

1 Introduction

There are some techniques can be used to solve the soliton equation with self-consistent sources [1–5]. Among this methods, the bilinear method and bilinear Bäcklund transformation have proved particularly powerful. Through the dependent variable transformations, some soliton equation with self-consistent sources can be transformed into bilinear forms. Applying the bilinear method developed by Hirota [6], we can obtain the soliton solutions. The construction of the bilinear Bäcklund transformation [7] by using Hirota method relies on a particular skill in using appropriate exchange formulas which are connected with the linear presentation of the system. Yet, the construct of bilinear Bäcklund transformation is complicated. Recently, Lambert, Gilson et al. [8–10] proposed an alternative procedure based on the use of the Bell polynomials which enable one to obtain parameter families of bilinear Bäcklund transformation and Lax pairs for the soliton equations in a lucid and systematic way. Fan et al. [11–14] have applied this method for some soliton equations. But there are not any research in the soliton equation with self-consistent sources.

In this paper, we will extend the Binary Bell polynomials to deal the Kadomtsev-Petviashvili equation with self-consistent sources (KPESCS). First, we derive the bilinear

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form for the KPESCS by the binary Bell polynomials. Second, the bilinear Bäcklund transformation and Lax pairs are obtained in a quick and natural manner.

2 The bilinear form for the mKP equation

The mail tool used here is a class of generalized multi-dimensional Bell polynomials. First, we give some notations on the Bell polynomials to easily understand our presentation.

Lambert et al. proposed a generalization of the Bell polynomial [8–10]. Let $n_k \geq 0$, $k=1,\cdots,l$, denote arbitrary integers, $f=f(x_1,\cdots,x_l)$ be a C^∞ multi-variable function, the following polynomials

$$\mathcal{Y}_{n_1x_1\cdots n_lx_l}(f)=\exp(-f)\partial_{x_1}^{n_1}\cdots\partial_{x_l}^{n_l}\exp(f) \tag{2.1}$$

is called multi-dimensional Bell polynomial (generalized Bell polynomial or $\mathcal{Y}$-polynomials). If all partial derivatives $f_{r_1x_1\cdots r_lx_l}=\partial_{x_1}^{r_1}\cdots\partial_{x_l}^{n_l}$ $(r_k=0,\cdots,n_k,\ k=1,\cdots l)$ are taken as different variable elements, then the generalized Bell polynomial $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)$ is the multivariable polynomial with respect to these variable elements $f_{r_1x_1,\cdots,r_lx_l}$. The subscripts in the notation $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)$ denote the highest order derivatives of f with respect to the variable x_k, $k=1,\cdots,l$ respectively.

For the special case $f=f(x,t)$, the associated two-dimensional Bell polynomials defined by (2.1) read

$$\mathcal{Y}_x(f)=f_x,\mathcal{Y}_{2x}(f)=f_{2x}+f_x^2, \qquad \mathcal{Y}_{3x}(f)=f_{3x}+3f_xf_{2x}+f_x^3, \tag{2.2a}$$

$$\mathcal{Y}_{x,t}(f)=f_{x,t}+f_xf_t, \qquad \mathcal{Y}_{2x,t}(f)=f_{2x,t}+f_{2x}f_t+2f_{x,t}f_x+f_x^2f_t. \tag{2.2b}$$

Base on the use of above Bell polynomials (2.1), the multidimensional binary Bell polynomials ($\mathcal{Y}$-polynomials) can be defined as follows

$$\begin{aligned}\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)=&\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(f)|_{f_{r_1x_1,\cdots,r_lx_l}}\\ =&\begin{cases} v_{r_1x_1,\cdots,r_lx_l}, & r_1+\cdots+r_l \text{ is odd},\\ w_{r_1x_1,\cdots,r_lx_l}, & r_1+\cdots+r_l \text{ is even},\end{cases}\end{aligned} \tag{2.3}$$

which is a multivariable polynomials with respect to all partial derivatives $v_{r_1x_1,\cdots,r_lx_l}(r_1+\cdots+r_l$ odd) and $w_{r_1x_1,\cdots,r_lx_l}(r_1+\cdots+r_l$ even), $r_k=0,\cdots,n_k$, $k=0,\cdots,l$.

The binary Bell polynomials also inherit the easily recognizable partial structure of the Bell polynomials. The lowest order binary Bell polynomials are

$$\mathcal{Y}_x(v)=v_x, \qquad \mathcal{Y}_{2x}(v,w)=w_{2x}+v_x^2, \tag{2.4a}$$

$$\mathcal{Y}_{x,t}(v,w)=w_{xt}+v_xv_t, \qquad \mathcal{Y}_{3x}(v,w)=v_{3x}+3v_xw_{2x}+v_x^3. \tag{2.4b}$$

The link between binary Bell polynomials $\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v,w)$ and the standard Hirota bilinear equation $D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G$ can be given by an identity

$$\mathcal{Y}_{n_1x_1,\cdots,n_lx_l}(v=\ln F/G,w=\ln FG)=(FG)^{-1}D_{x_1}^{n_1}\cdots D_{x_l}^{n_l}F\cdot G, \tag{2.5}$$