Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **8**, No. 2, pp. 293-305

DOI: 10.4208/aamm.2014.m798
April 2016

Exact Solutions for Fractional Differential-Difference Equations by (G'/G)-Expansion Method with Modified Riemann-Liouville Derivative

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Received 14 October 2014; Accepted (in revised version) 21 April 2015

Abstract. In this paper, the (G'/G)-expansion method is suggested to establish new exact solutions for fractional differential-difference equations in the sense of modified Riemann–Liouville derivative. The fractional complex transform is proposed to convert a fractional partial differential difference equation into its differential difference equation of integer order. With the aid of symbolic computation, we choose nonlinear lattice equations to illustrate the validity and advantages of the algorithm. It is shown that the proposed algorithm is effective and can be used for many other nonlinear lattice equations in mathematical physics and applied mathematics.

AMS subject classifications: 34A08, 35R11, 37H10, 26A33, 35Q68

Key words: (G'/G)-expansion method, fractional Hybrid Lattice equation, fractional modified Korteweg-de Vries equation, modified Riemann-Liouville derivative.

1 Introduction

Since the study of Fermi, Pasta, and Ulam in the 1960s [1], nonlinear differential-difference equations (NLDDEs) have been the focus of many nonlinear studies and much attention have been paid to the research of the theory of differential-difference equations during the last decades [2–5]. Among these research works, the investigation of exact solutions of nonlinear differential difference equations plays an important role in the study of

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nonlinear physical phenomena. It is well known that NLDDEs are often presented to describe the motion of the isolated waves, localized in a small part of space, in many fields such as hydrodynamic, physics, biophysics, plasma physics, molecular crystals and nonlinear optic. And in the past several decades, many effective methods for obtaining exact solutions of NLDDEs have been presented [6–11].

Fractional differential equations are generalizations of classical differential equations of integer order. In recent decades, fractional differential equations have been the focus of many studies due to their frequent appearance in various applications in physics, biology, engineering, signal processing, systems identification, electrochemistry, finance and fractional dynamics. The fractional differential equations have been investigated by many researchers [12, 13]. Many powerful methods for solving nonlinear fractional ordinary differential equations and fractional partial differential equations were appeared in open literature [14–23].

In the literature, on the basis of the problem under investigation, there exist vast different definitions of fractional derivatives such as Grünwald–Letnikov derivative, Riemann–Liouville derivative, Caputo derivative, modified Riemann–Liouville derivative and these derivatives are widely used. The fractional complex transform [24, 25] is used to analytically deal with fractional differential equations. This method extremely simple but effective for solving fractional differential equations.

In this article, based on a fractional complex transformation, a given fractional differential difference equation in the sense of modified Riemann–Liouville derivative can be turned into differential difference equation of integer order, and the reduced equations can be solved by symbolic computation. The manuscript suggests the (G'/G)-expansion method and fractional complex transform to find the exact solutions of nonlinear differential difference equations with the modified Riemann-Liouville derivative by Jumarie [26]. The Jumarie's modified Riemann–Liouville derivative of order α is defined by the following expression:

$$D_t^\alpha f(t)=\begin{cases} \dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dt}\displaystyle\int_0^t (t-\xi)^{-\alpha}(f(\xi)-f(0))d\xi, & 0<\alpha<1, \\ (f^{(n)}(t))^{(\alpha-n)}, & n\le\alpha<n+1, \quad n\ge 1. \end{cases} \tag{1.1}$$

Modified Riemann–Liouville derivative has many important properties, one of them is the α order derivative constant is zero and four famous formulas of them are [27, 28]

$$D_t^\alpha x^\gamma=\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)}x^{\gamma-\alpha}, \quad \gamma>0, \tag{1.2a}$$

$$D_t^\alpha(cf(t))=cD_t^\alpha f(t), \quad c=\text{constant}, \tag{1.2b}$$

$$D_t^\alpha\{af(t)+bg(t)\}=aD_t^\alpha f(t)+bD_t^\alpha g(t), \tag{1.2c}$$

$$D_x^\alpha c=0, \quad c=\text{constant}, \tag{1.2d}$$

which are direct consequences of the equality

$$d^\alpha x(t)=\Gamma(1+\alpha)dx(t), \tag{1.3}$$