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The Motion of a Neutrally Buoyant Ellipsoid Inside Square Tube Flows

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Abstract. The motion and rotation of an ellipsoidal particle inside square tubes and rectangular tubes with the confinement ratio $R/a \in (1.0,4.0)$ are studied by the lattice Boltzmann method (LBM), where R and a are the radius of the tube and the semi-major axis length of the ellipsoid, respectively. The Reynolds numbers (Re) up to 50 are considered. For the prolate ellipsoid inside square and rectangular tubes, three typical stable motion modes which depend on R/a are identified, namely, the kayaking mode, the tumbling mode, and the log-rolling mode are identified for the prolate spheroid. The diagonal plane strongly attracts the particle in square tubes with $1.2 \leq R/a < 3.0$. To explore the mechanism, some constrained cases are simulated. It is found that the tumbling mode in the diagonal plane is stable because the fluid force acting on the particle tends to diminish the small displacement and will bring it back to the plane. Inside rectangular tubes the particle will migrate to a middle plane between short walls instead of the diagonal plane. Through the comparisons between the initial unstable equilibrium motion state and terminal stable mode, it is seems that the particle tend to adopt the mode with smaller kinetic energy.

AMS subject classifications: 65M99, 76D99

Key words: LBM, Poiseuille flow, square tubes, ellipsoidal particles.

1 Introduction

The motion of the particles in tubes are ubiquitous in nature and many applications in industries, such as chemical, biological, and mechanical engineering. Many studies on the motion of particles in simple flows have been carried out, for example, the particle's rotational behaviors in Couette flow [1–6], sedimentation of particles inside tubes [7]. Here we focus on the particle's behaviors in tube flows.

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In 1961, Segre and Silberberg studied the migration of neutrally buoyant spherical particles in Poiseuille flows experimentally and found that the particles migrate towards an equilibrium position and equilibrate at a distance of 0.6 time the radius of the tube from the tube's center [8]. In one hand, the particle experiences the "Magnus effect" due to the rotation of the particle. The rotation of the particle is induced by the shear stress in the Poiseuille flow. When the particle migrates radially to the wall, the fluid between the wall and the particle is squeezed and inversely, the particle will experience high pressure to prevent it to reach the wall. Hence, the particle will seek an equilibrium position between the axis and the wall where the total radial force is zero. Karnis et al. [9] studied the migration of non-spherical particles in tubes in 1966. They observed that for a rod-like particle, the major axis of the particle rotates on the plane passing though the center of the particle and the tube axis (tumbling state), while for a disk-like particle, it rotates with its minor axis on the same plane (log-rolling state).

Feng and Joseph simulated the motion of a single ellipse in two-dimensional (2D) creeping flows using a Finite Element (FE) method [10]. According to their study, a neutrally buoyant particle exhibits the Segre-Silberberg effect in a Poiseuille flow. The driving forces of the migration have been identified as a wall repulsion due to lubrication, an inertial lift related to shear slip, a lift due to particle rotation and the velocity profile curvature. By examining the pressure and shear stress distributions on the particle, they found that the stagnation pressure on the particle surface are particularly important in determining the direction of migration.

Yang et al. [11] simulated a single neutrally buoyant spherical particle in tube flows and they used a method of constrained simulation to obtain correlation formulas for the lift force, slip velocity, and equilibrium position.

Sugihara-Seki numerically studied the motions of an inertialess elliptical particle in tube Poiseuille flow using a Finite Element (FE) method [12]. A prolate spheroid is found to either tumble or oscillate in rotation, depending on the particle-tube size ratio, the axis ratio of the particle, and the initial conditions. A large oblate spheroid may approach asymptotically a steady, stable slightly inclined configuration, at which it is located close to the tube centreline.

However, in the paper they consider only the motion where two of the three principal axes of the ellipsoid lie in a plane containing the tube axis and the fluid motion is assumed to be symmetric with respect to this plane. On the other hand, the inertia of the particle is very important in this flow problem but it is not considered. Hence it is only a starting point for the analysis of the general motion of an ellipsoid in tube flows.

Yu et al. studied the the migration in a Poiseuille flow using a finite-difference-based DLM method [13]. They found that suppression of the sphere rotation produces significant large additional lift forces pointing towards the tube axis on the spheres in the neutrally buoyant cases.

Pan et al. [14] simulated the motion of a neutrally buoyant ellipsoid in a Poiseuille flow and investigated its rotational and orientational behavior inside circular tubes. However, the study only considered circular tubes with fixed $R/a \approx 3$.