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Viscosity and Diffusion Effects at the Boundary Surface of Viscous Fluid and Thermoelastic Diffusive Solid Medium

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Abstract. This paper concentrates on the wave motion at the interface of viscous compressible fluid half-space and homogeneous isotropic, generalized thermoelastic diffusive half-space. The wave solutions in both the fluid and thermoelastic diffusive half-spaces have been investigated; and the complex dispersion equation of leaky Rayleigh wave motion have been derived. The phase velocity and attenuation coefficient of leaky Rayleigh waves have been computed from the complex dispersion equation by using the Muller's method. The amplitudes of displacements, temperature change and concentration have been obtained. The effects of viscosity and diffusion on phase velocity and attenuation coefficient of leaky Rayleigh waves motion for different theories of thermoelastic diffusion have been depicted graphically. The magnitude of heat and mass diffusion flux vectors for different theories of thermoelastic diffusion have also been computed and represented graphically.

AMS subject classifications: 35Q80, 37D35, 74F05, 76N17, 80A17

Key words: Viscous compressible fluid, isotropic, thermoelastic diffusion, phase velocity, attenuation coefficient.

1 Introduction

The problem of fluid-structure interaction is wide and covers many types of both fluid and structural behavior. Such problems can be interesting for researching processes of vibrodisplacement and localization, decontamination of liquid medium, airing and dispersion; in bioacoustics and cardiovascular medicine (for instance, for some problems involving blood flow, where fluid and structures models are coupled); in nondestructive testing (for instance, the scattering of acoustic waves can give important information about the internal composition of solids and fluids, yielding information about internal inhomogeneities, asymmetries and defects from the scattering system);

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in technologies of resumption of oil production in foul wells, etc. For many years, numerous authors have been interested in dynamics of fluid-structure interaction both for unbounded domains of either fluid or structure and for delimited ones, involving both motionless and flowing fluids.

Sorokin and Terentiev [26] investigated the effects of generation and transmission of the vibro-acoustic energy in an elastic cylindrical shell filled with water. Sorokin [27] discussed the assessment of the validity of elementary models of wave propagation in an isotropic elastic layer under heavy fluid loading as well as analysis of coupling effects due to uneven fluid loading. Sorokin and Chubinskij [28] studied free wave propagation and attenuation in elastic plates loaded by a quiescent viscous compressible fluid. Ashour [6] discussed wave motion in a viscous fluid-filled fracture. Hasheminejad and Safari [12, 13] studied dynamic viscoelastic effects on sound wave diffraction by spherical and cylindrical shells submerged in and filled with viscous compressible fluids and acoustic scattering from viscoelastically coated spheres and cylinders in viscous fluids. Nayfeh and Nagy [17] investigated the problem of excess attenuation of leaky Lamb waves due to viscous fluid loading in which they also studied the propagation of leaky rayleigh waves at the interface of viscous compressible fluid half-space and homogeneous isotropic elastic half-space.

The classical theory of thermoelasticity is based on the Fourier's heat conduction theory which assumes that the thermal disturbances propagate at infinite speed. This prediction is unrealistic from the physical point of view, particularly in situations like those involving very short transient durations, sudden high heat flux exposures, and at very low temperatures near the absolute zero. In the last two decades, two different generalizations of the classical theory of thermoelasticity were developed which predict the so-called second-sound effects, that is, which predict only finite velocity of propagation for heat and displacement fields. Lord and Shulman [16] incorporated a flux rate term into the Fourier's law of heat conduction and formulated a generalized theory admitting finite speed for thermal signals. Green and Lindsay [11] have developed a temperature rate dependent thermoelasticity by including temperature rate among the constitutive variables, which does not violate the classical Fourier's law of heat conduction when the body under consideration has a center of symmetry, and this theory also predicts a finite speed of heat propagation. The Lord and Shulman [16] theory of generalized thermoelasticity was further extended to homogeneous anisotropic heat conducting materials recommended by Dhaliwal and Sherief [8]. Chanderashekhariah [7] refers to this wave like thermal disturbance as second sound. A survey article of various representative theories in the range of generalized thermoelasticity have been brought out by Hetnarski and Ignaczak [14].

The spontaneous movement of the particles from high concentration region to the low concentration region is defined as diffusion and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. The thermodiffusion in elastic solids is due to coupling of fields of temperature, mass diffusion and that of strain in addition to heat and mass exchange with environment. Thermal diffusion utilizes the transfer of heat across a thin liquid or gas

to accomplish isotope separation. Today, thermal diffusion remains a practical process to separate isotopes of noble gases (e.g., xexon) and other light isotopes (e.g., carbon) for research purposes. In integrated circuit fabrication diffusion is used to introduce dopants in controlled amounts into the semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, and the source/drain regions in Metal Oxide Semiconductor (MOS) transistors and dope poly-silicon gates in MOS transistors. In most of the applications, the concentration is calculated using what is known as Fick's law. This is a simple law which does not take into consideration the mutual interaction between the introduced substance and the medium into which it is introduced or the effect of temperature on this interaction. Study of phenomenon of diffusion is used to improve the conditions of oil extraction (seeking ways of more efficiently recovering oil from oil deposits). These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits.

Nowacki [18–21] developed the theory of thermoelastic diffusion by using coupled thermoelastic model. Sherief and Saleh [23] investigated the problem of a thermoelastic half-space in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Singh discussed the reflection phenomena of waves from free surface of an thermoelastic diffusion elastic solid with one relaxation time in [24] and with two relaxation times in [25]. Aouadi studied in [1] the generalized thermoelastic diffusion problem with variable electrical and thermal conductivity. Aouadi studied also the interaction between the processes of elasticity, heat and diffusion in an infinitely long solid cylinder [2] and in an infinite elastic body with spherical cavity [3]. Gawinecki and Szymaniec [9] proved a theorem about global existence of the solution for a nonlinear parabolic thermoelastic diffusion problem. Gawinecki et al. [10] proved a theorem about existence, uniqueness and regularity of the solution for the same problem. Uniqueness and reciprocity theorems for the equations of generalized thermoelastic diffusion problem, in isotropic media, was proved by Sherief et al. [22] on the basis of the variational principle equations, under restrictive assumptions on the elastic coefficients. Due to the inherit complexity of the derivation of the variational principle equations, Aouadi [4] proved this theorem in the Laplace transform domain, under the assumption that the functions of the problem are continuous and the inverse Laplace transform of each is also unique. Recently, Aouadi [5] derived the uniqueness and reciprocity theorems for the generalized problem in anisotropic media, under the restriction that the elastic, thermal conductivity and diffusion tensors are positive definite. Kumar and Kansal [15] developed the basic equation of anisotropic thermoelastic diffusion based upon Green-Lindsay model.

The present article is aimed to study the propagation of leaky Rayleigh waves in a viscous compressible fluid half-space overlying a homogeneous isotropic, thermoelastic diffusive half-space in the context of generalized theories of thermoelastic diffusion. The phase velocity, attenuation coefficients of leaky Rayleigh wave propagation have been computed from the complex dispersion equation by using the Muller's method. The amplitudes of displacements, temperature change and concentration

have been obtained. The magnitude of heat and mass diffusion flux vectors for different theories of thermoelastic diffusion have also been computed and represented graphically.

2 Basic equations

Following Sherief et al. [22] and Kumar et al. [15], the basic equations for homogeneous isotropic generalized thermoelastic diffusion in the absence of body forces, heat and mass diffusion sources are:

Constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}\left[\lambda e_{kk} - \beta_1(T + \tau_1\dot{T}) - \beta_2(C + \tau^1\dot{C})\right], \tag{2.1a}$$

$$\rho T_0 S = k + \rho C_E(T + \alpha\dot{T}) + \beta_1 T_0 e_{kk} + aT_0(C + \beta\dot{C}), \tag{2.1b}$$

$$P = -\beta_2 e_{kk} + b(C + \tau^1\dot{C}) - a(T + \tau_1\dot{T}); \tag{2.1c}$$

Equations of motion

$$\mu u_{i,jj} + (\lambda + \mu)u_{j,ij} - \beta_1(T + \tau_1\dot{T})_{,i} - \beta_2(C + \tau^1\dot{C})_{,i} = \rho\ddot{u}_i; \tag{2.2}$$

Equation of heat conduction

$$\rho C_E(\dot{T} + \tau_0\ddot{T}) + \beta_1 T_0(\dot{e}_{kk} + \varepsilon\tau_0\ddot{e}_{kk}) + aT_0(\dot{C} + \gamma\ddot{C}) = KT_{,ii}; \tag{2.3}$$

Equation of mass diffusion

$$D\beta_2 e_{kk,ii} + Da(T + \tau_1\dot{T})_{,ii} + (\dot{C} + \varepsilon\tau^0\ddot{C}) - Db(C + \tau^1\dot{C})_{,ii} = 0; \tag{2.4}$$

where

$$\beta_1 = (3\lambda + 2\mu)\alpha_t, \quad \text{and} \quad \beta_2 = (3\lambda + 2\mu)\alpha_c,$$

λ, μ are Lame's constants, α_t is the coefficient of linear thermal expansion and α_c is the coefficient of linear diffusion expansion. a, b are, respectively, coefficients describing the measure of thermoelastic diffusion effects and of diffusion effects, T_0 is the reference temperature assumed to be such that $|T/T_0| \ll 1$. σ_{ij} are the components of the stress tensor. u_i are the components of the displacement vector $\mathbf{u}$, ρ is the density assumed to be independent of the time, e_{ij} are the components of the strain tensor, S is the entropy per unit mass, P is the chemical per unit mass, $T(x_1, x_2, x_3, t)$ is the temperature change, C is the concentration, C_E is the specific heat at the constant strain, K is the coefficient of the thermal conductivity, D is the thermoelastic diffusion constant. τ^0, τ^1 are diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$, and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here

$$\alpha = \beta = \varepsilon = \gamma = k = \tau_0 = \tau^0 = \tau_1 = \tau^1 = 0,$$

for Coupled Thermoelasticity (CT) model,

$$\alpha = \beta = k = \tau_1 = \tau^1 = 0, \quad \varepsilon = 1, \quad \gamma = \tau_0,$$

for Lord-Shulman (L-S) model, and

$$\alpha = \tau_0, \quad \beta = \tau^0, \quad \varepsilon = 0, \quad \gamma = \tau^0,$$

for Green-Lindsay (G-L) model. Following Sorokin et al. [28], the linearlised Navier-Stokes equations of motion of viscous compressible fluid are:

Equations of momentum

$$\rho_f \frac{\partial \vec{v}}{\partial t} = -\nabla p + \frac{\eta}{3}\nabla(\nabla \cdot \vec{v}) + \eta \nabla^2 \vec{v}; \tag{2.5}$$

Equation of continuity

$$\frac{\partial \rho^0}{\partial t} + \rho_f \nabla \cdot \vec{v} = 0; \tag{2.6}$$

The constitutive law

$$\frac{\partial p}{\partial \rho^0} = c_f^2. \tag{2.7}$$

Here, $\vec{v}=(v_1, v_2, v_3)$ is the velocity vector, p is the fluid pressure, ρ^0 is the density pertubation, ρ_f is the fluid density at rest, $c_f=\sqrt{K_f/\rho_f}$ is the sound velocity, K_f is the bulk modulus, ν and $\eta=\rho_f\nu$ are the kinematic and dynamic fluid viscosity, respectively. The stress tensor π_{ij} in the fluid is

$$\pi_{ij} = \Big(-p - \frac{2}{3}\eta \nabla \cdot \vec{v} \Big)\delta_{ij} + 2\eta \epsilon_{ij}. \tag{2.8}$$

Here, δ_{ij} is a unit tensor, $\epsilon_{ij}=(v_{i,j} + v_{j,i})/2$ is the tensor of velocities of deformations.

3 Formulation of the problem

We consider viscous compressible fluid half-space (Medium II) overlying a homogeneous isotropic, generalized thermoelastic diffusive half-space (Medium I). The origin of the cartesian coordinate system (x, y, z) is taken at any point on the plane surface (interface) and z-axis points vertically downwards into the solid half-space which is thus represented by $z \geq 0$ (see Fig. 1). We chose x-axis in the direction of wave propagation so that all particles on a line parallel to y-axis are equally displaced. Therefore all the field quantities are independent of y. Therefore for two dimensional problem,

$$\overrightarrow{u}(x, z, t) = (u, 0, w), \quad T(x, z, t), \quad C(x, z, t), \quad \overrightarrow{v}(x, z, t) = (v_1, 0, v_3), \tag{3.1}$$

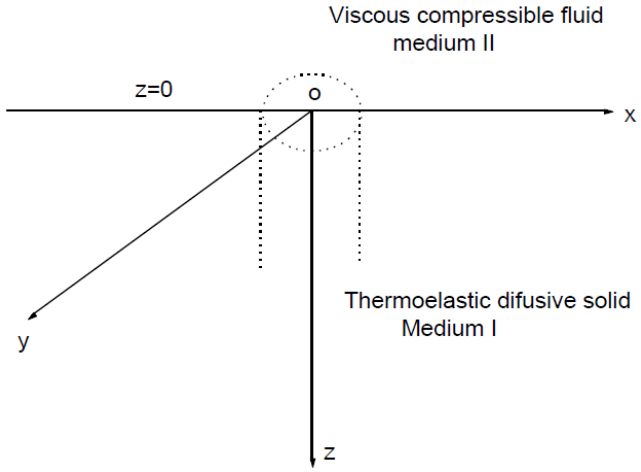

Figure 1: Geometry of the problem.

are displacement vector, temperature change, concentration and velocity vector. We define the dimensionless quantities:

$$x' = \frac{w_1^* x}{c_1}, \quad z' = \frac{w_1^* z}{c_1}, \quad t' = w_1^* t, \quad u' = \frac{w_1^* u}{c_1}, \quad w' = \frac{w_1^* w}{c_1}, \tag{3.2a}$$

$$T' = \frac{\beta_1 T}{\rho c_1^2}, \quad C' = \frac{\beta_2 C}{\rho c_1^2}, \quad P' = \frac{P}{\beta_2}, \quad \tau_0' = w_1^* \tau_0, \quad \tau_1' = w_1^* \tau_1, \tag{3.2b}$$

$$\tau^{0'} = w_1^* \tau^0, \quad \tau^{1'} = w_1^* \tau^1, \quad \vec{v}' = \frac{\vec{v}}{c_1}, \quad p' = \frac{p}{\beta_1 T_0}, \quad \rho_f^* = \frac{\rho_f c_1^2}{\beta_1 T_0}, \tag{3.2c}$$

$$\rho^{0'} = \frac{\rho^0 c_1^2}{\beta_1 T_0}, \quad \eta^* = \frac{\eta w_1^*}{\rho_f c_1^2}, \quad \eta_1^* = \frac{\eta w_1^*}{\beta_1 T_0}, \quad \sigma_{ij}' = \frac{\sigma_{ij}}{\beta_1 T_0}, \quad \pi_{ij}' = \frac{\pi_{ij}}{\beta_1 T_0}, \tag{3.2d}$$

$$w_1^* = \frac{\rho C_E c_1^2}{K}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}, \quad \delta^2 = \frac{c_2^2}{c_1^2}. \tag{3.2e}$$

Here w_1^* is the characteristic frequency of the Medium I, c_1, c_2 are the longitudinal and transverse wave velocities in the Medium I. Upon introducing the quantities Eq. (3.2) in Eqs. (2.2)-(2.4) with the aid of Eq. (3.1) and after suppressing the primes, we obtain

$$(1-\delta^2)\nabla(\nabla\cdot\vec{u}) + \delta^2\nabla^2\vec{u} - \tau_t^1\nabla T - \tau_c^1\nabla C = \ddot{\vec{u}}, \tag{3.3a}$$

$$\nabla^2 T = \tau_t^0 \dot{T} + \zeta_1 \tau_c^0 \dot{C} + \zeta_2 \tau_e^0 \dot{e}, \tag{3.3b}$$

$$q_1^* \nabla^2 e + q_2^* \tau_t^1 \nabla^2 T - q_3^* \tau_c^1 \nabla^2 C + \tau_f^0 \dot{C} = 0, \tag{3.3c}$$

where

$$\zeta_1 = \frac{aT_0 c_1^2 \beta_1}{w_1^* K \beta_2}, \quad \zeta_2 = \frac{\beta_1^2 T_0}{\rho K w_1^*}, \quad q_1^* = \frac{D w_1^* \beta_2^2}{\rho c_1^4}, \quad q_2^* = \frac{D w_1^* \beta_2 a}{\beta_1 c_1^2}, \quad q_3^* = \frac{D w_1^* b}{c_1^2},$$

$$\tau_t^1 = 1 + \tau_1 \frac{\partial}{\partial t}, \quad \tau_c^1 = 1 + \tau^1 \frac{\partial}{\partial t}, \quad \tau_t^0 = 1 + \tau_0 \frac{\partial}{\partial t}, \quad \tau_c^0 = 1 + \gamma \frac{\partial}{\partial t},$$

$$\tau_e^0 = 1 + \varepsilon\tau_0 \frac{\partial}{\partial t}, \quad \tau_f^0 = 1 + \varepsilon\tau^0 \frac{\partial}{\partial t}, \quad e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}, \quad \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Using Eq. (3.2) in Eqs. (2.5)-(2.7), we obtain

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_f^*}\nabla p + \frac{\eta^*}{3}\nabla(\nabla \cdot \vec{v}) + \eta^*\nabla^2\vec{v}, \tag{3.4a}$$

$$\frac{\partial \rho^0}{\partial t} + \rho_f^*\nabla \cdot \vec{v} = 0, \tag{3.4b}$$

$$\frac{\partial p}{\partial \rho^0} = \frac{c_f^2}{c_1^2}. \tag{3.4c}$$

Straightforward transformation of Eqs. (3.4a) and (3.4c) with the aid of Eq. (3.4b) yield

$$\frac{\partial \vec{v}}{\partial t} - \eta^*\nabla^2\vec{v} + \frac{1}{\rho_f^*}\Big(1 + \frac{\eta^* c_1^2}{3c_f^2}\frac{\partial}{\partial t}\Big)\nabla p = 0. \tag{3.5}$$

For thermoelastic diffusive half-space, we introduce the potential functions ϕ and ψ through the relations

$$u = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}, \tag{3.6}$$

where ϕ and ψ are the displacement potentials of longitudinal and shear waves. For viscous fluid, we have

$$v_1 = \frac{\partial \Phi}{\partial x} - \frac{\partial \Psi}{\partial z}, \qquad v_3 = \frac{\partial \Phi}{\partial z} + \frac{\partial \Psi}{\partial x}, \tag{3.7}$$

where Φ and Ψ are the respectively scalar velocity and vector velocity potential components. Using Eq. (3.6) in Eqs. (3.3a)-(3.3c), we obtain

$$\nabla^2\phi - \tau_t^1 T - \tau_c^1 C = \ddot{\phi}, \tag{3.8a}$$

$$\nabla^2\psi - \frac{\ddot{\psi}}{\delta^2} = 0, \tag{3.8b}$$

$$\nabla^2 T = \tau_t^0\dot{T} + \zeta_1\tau_c^0\dot{C} + \zeta_2\tau_e^0\nabla^2\dot{\phi}, \tag{3.8c}$$

$$q_1^*\nabla^4\phi + q_2^*\tau_t^1\nabla^2 T - q_3^*\tau_c^1\nabla^2 C + \tau_f^0\dot{C} = 0. \tag{3.8d}$$

Similarly, using Eq. (3.7) in Eq. (3.5), we get

$$\Big(1 + \frac{4}{3}\frac{\eta^* c_1^2}{c_f^2}\frac{\partial}{\partial t}\Big)\nabla^2\Phi - \frac{c_1^2}{c_f^2}\frac{\partial^2\Phi}{\partial t^2} = 0, \tag{3.9a}$$

$$\eta^*\nabla^2\Psi - \frac{\partial \Psi}{\partial t} = 0, \tag{3.9b}$$

where, the pressure p is given by the relation

$$p = \rho_f^*\Big(\frac{4}{3}\eta^*\nabla^2 - \frac{\partial}{\partial t}\Big)\Phi. \tag{3.10}$$

4 Solution of the problem

We assume solutions of Eqs. (3.8a)-(3.9b) of the form

$$(\phi,\psi,T,C,\Phi,\Psi)=\left[f(z),g(z),w(z),h(z),\bar{\Phi}(z),\bar{\Psi}(z)\right]e^{\iota\xi(x-ct)}, \tag{4.1}$$

where $c=\omega/\xi$ is the dimensionless phase velocity, ω is the frequency, ξ is the complex wave number. Using Eq. (4.1) in Eqs. (3.8a)-(3.9b) and solving the resulting differential equations, the expressions for ϕ,ψ,T,C,Φ and Ψ are obtained as

$$\phi=(A_1e^{-\xi m_1z}+A_2e^{-\xi m_2z}+A_3e^{-\xi m_3z})e^{\iota\xi(x-ct)}, \tag{4.2a}$$
$$T=(n_1A_1e^{-\xi m_1z}+n_2A_2e^{-\xi m_2z}+n_3A_3e^{-\xi m_3z})e^{\iota\xi(x-ct)}, \tag{4.2b}$$
$$C=(k_1A_1e^{-\xi m_1z}+k_2A_2e^{-\xi m_2z}+k_3A_3e^{-\xi m_3z})e^{\iota\xi(x-ct)}, \tag{4.2c}$$
$$\psi=A_4e^{-\xi m_4z}e^{\iota\xi(x-ct)}, \tag{4.2d}$$
$$\Phi=A_5e^{\xi m_5z}e^{\iota\xi(x-ct)}, \tag{4.2e}$$
$$\Psi=A_6e^{\xi m_6z}e^{\iota\xi(x-ct)}, \tag{4.2f}$$

where m_i^2, $i=1,2,3$ are the roots of polynomial equation:

$$m^6+A^*m^4+B^*m^2+C^*=0, \tag{4.3}$$

where

$$\tau_t^{11}=1-\iota\omega\tau_1,\qquad \tau_c^{11}=1-\iota\omega\tau^1,\qquad \tau_t^{10}=-\iota\omega(1-\iota\omega\tau_0),$$
$$\tau_c^{10}=-\iota\omega(1-\iota\omega\gamma),\quad \tau_e^{10}=-\iota\omega(1-\iota\omega\varepsilon\tau_0),\quad \tau_f^{10}=-\iota\omega(1-\iota\omega\varepsilon\tau^0),$$
$$r_1=(q_1^*-q_3^*)\tau_c^{11},$$
$$r_2=\tau_f^{10}-(q_1^*+q_2^*)(\tau_t^{10}\tau_c^{11}-\zeta_1\tau_c^{10}\tau_t^{11})+(q_2^*+q_3^*)(\tau_t^{10}+\zeta_2\tau_t^{11}\tau_e^{10})\tau_c^{11}-q_3^*\tau_c^{11}\omega^2,$$
$$r_3=\tau_f^{10}(\omega^2-\tau_t^{10}-\zeta_2\tau_t^{11}\tau_e^{10})+\omega^2(q_2^*+q_3^*)\tau_t^{10}\tau_c^{11}-q_2^*\omega^2(\tau_t^{10}\tau_c^{11}-\zeta_1\tau_c^{10}\tau_t^{11}),$$
$$r_4=c^2\tau_f^{10}\tau_t^{10},$$
$$A^*=\frac{-3\xi^2r_1+r_2}{r_1\xi^2},\quad B^*=\frac{3\xi^4r_1-2\xi^2r_2+r_3}{r_1\xi^4},\quad C^*=\frac{-\xi^4r_1+\xi^2r_2-r_3-r_4}{r_1\xi^4},$$
$$\Delta_{1i}=(-q_3^*\tau_c^{11})\xi^4m_i^4+(q_3^*\tau_c^{11}(\tau_t^{10}+2\xi^2)+\tau_f^{10}+q_2^*\zeta_1\tau_t^{11}\tau_c^{10})\xi^2m_i^2$$
$$-(\xi^2+\tau_t^{10})(q_3^*\tau_c^{11}\xi^2+\tau_f^{10})-q_2^*\xi^2\zeta_1\tau_t^{11}\tau_c^{10},$$
$$\Delta_{2i}=(q_1^*\zeta_1\tau_c^{10}+q_3^*\tau_c^{11}\zeta_2\tau_e^{10})\xi^4m_i^4-(2\xi^2q_1^*\zeta_1\tau_c^{10}+\zeta_2\tau_f^{10}\tau_e^{10}+2q_3^*\xi^2\tau_c^{11}\zeta_2\tau_e^{10})\xi^2m_i^2$$
$$+q_1^*\zeta_1\tau_c^{10}\xi^4+\zeta_2\tau_e^{10}\xi^2(\tau_f^{10}+q_3^*\tau_c^{11}\xi^2),$$
$$\Delta_{3i}=-q_1^*\xi^6m_i^6+(3q_1^*\xi^2-q_2^*\tau_t^{11}\zeta_2\tau_e^{10}+q_1^*\tau_t^{10})\xi^4m_i^4-(3q_1^*\xi^2-2\zeta_2q_2^*\tau_t^{11}\tau_e^{10}$$
$$+2q_1^*\tau_t^{10})\xi^4m_i^2+\xi^4\left(q_1^*(\xi^2+\tau_t^{10})-\zeta_2q_2^*\tau_t^{11}\tau_e^{10}\right),$$
$$n_i=-\frac{\Delta_{2i}}{\Delta_{1i}},\quad k_i=\frac{\Delta_{3i}}{\Delta_{1i}},\quad i=1,2,3.$$

The roots m_i^2, $i = 4,5,6$ are given as

$$m_4^2 = 1 - \frac{c^2}{\delta^2}, \quad m_5^2 = 1 - \frac{c_1^2 c^2}{c_f^2 - 4\iota\omega\eta^* c_1^2/3}, \quad m_6^2 = 1 - \frac{\iota c}{\eta^* \xi}.$$

5 Boundary conditions

The boundary conditions at the solid-fluid interface $z = 0$ to be satisfied are as follows:

(i) Mechanical conditions

$$\sigma_{zz} = \pi_{zz}, \qquad \sigma_{xz} = \pi_{xz}, \tag{5.1}$$

(ii) Velocity conditions

$$\frac{\partial u}{\partial t} = \nu_1, \qquad \frac{\partial w}{\partial t} = \nu_3, \tag{5.2}$$

(iii) Thermal condition

$$\frac{\partial T}{\partial z} = 0, \tag{5.3}$$

(iv) Concentration condition

$$\frac{\partial C}{\partial z} = 0. \tag{5.4}$$

6 Derivation of secular equations

Invoking the boundary conditions Eqs. (5.1)-(5.4) on the surface z=0 and using Eqs. (4.2a)-(4.2f) and solving a system of six simultaneous equations, we obtain the complex dispersion equation at the interface of viscous compressible fluid half-space and homogeneous isotropic, generalized thermoelastic diffusive half-space as:

$$R_1 S_1 + \iota R_2(\mu^* S_2 - \omega S_3) + \omega R_3 S_4 = 0, \tag{6.1}$$

where

$$\lambda^* = \frac{\lambda}{\beta_1 T_0}, \qquad \mu^* = \frac{\mu}{\beta_1 T_0},$$
$$d_i = -\lambda^* + (\lambda^* + 2\mu^*)(m_i^2 - \tau_t^{11} n_i \xi^{-2} - \tau_c^{11} k_i \xi^{-2}), \quad i = 1,2,3,$$
$$R_1 = (k_2 n_1 - n_2 k_1)(n_1 m_1 d_3 - n_3 m_3 d_1) m_2 - (k_3 n_1 - n_3 m_1)(n_1 m_1 d_2 - n_2 m_2 d_1) m_3,$$
$$R_2 = m_1 m_2 m_3 [(k_2 n_1 - n_2 k_1)(n_1 - n_3) - (k_3 n_1 - n_3 k_1)(n_1 - n_2)],$$
$$R_3 = (k_2 n_1 - n_2 k_1)(n_1 m_1 - n_3 m_3) m_2 - (k_3 n_1 - n_3 m_1)(n_1 m_1 - n_2 m_2) m_3,$$
$$S_1 = -\frac{\mu^*}{2}(m_4^2 + 1)(m_5 m_6 - 1) - \iota\omega\eta_1^* \Big(m_5 m_6 - \frac{1}{2}(m_6^2 + 1)\Big) + \frac{\iota\omega\eta_1^* m_4 m_5}{2}(m_6^2 - 1),$$
$$S_2 = -2\iota\mu^* m_4 (m_5 m_6 - 1) + 2\omega m_4 \eta_1^* \Big(m_5 m_6 - \frac{1}{2}(m_6^2 + 1)\Big) - \omega\eta_1^* m_6 (m_6^2 - 1),$$

$$S_3 = -2\mu^*\eta_1^* m_4\Big(m_5 m_6 - \frac{1}{2}(m_6^2+1)\Big) - \frac{\mu^*\eta_1^* m_6}{2}(m_4^2+1)(m_6^2-1)$$
$$- \frac{\iota\omega\eta_1^* m_4}{2}(4m_5 m_6\eta_1^* - (m_6^2+1)^2\eta_1^*),$$
$$S_4 = -\iota\mu^*\eta_1^* m_4 m_5(m_6^2-1) - \iota\mu^*\eta_1^*(m_4^2+1)\Big(m_5 m_6 - \frac{1}{2}(m_6^2+1)\Big)$$
$$+ \frac{\omega\eta_1^*}{2}(4m_5 m_6\eta_1^* - (m_6^2+1)^2\eta_1^*).$$

6.1 Particular cases

In the absence of diffusion effect, i.e., if we take a=b=β_2= 0 in Eq. (6.1), we obtain the corresponding complex dispersion equation at the interface of viscous compressible fluid half-space and homogeneous isotropic generalized thermoelastic half-space as

$$(n_1' f_1 h_2 - n_2' f_2 h_1)S_1 + \iota f_1 f_2(n_1' - n_2')(\mu^* S_2 - \omega S_3) + \omega(n_1' f_1 - n_2' f_2)S_4 = 0, \quad (6.2)$$

where

$$E = -2\xi^2 + \omega^2 - \tau_t^{10} - \tau_t^{11}\zeta^2\tau_e^{10}, \qquad F = \xi^2 - (\omega^2 - \tau_t^{10} - \tau_t^{11}\zeta^2\tau_e^{10}) - \tau_t^{10}c^2,$$
$$f_i^2 = \frac{-E \pm \sqrt{E^2 - 4F\xi^2}}{2\xi^2}, \qquad n_i' = -\frac{\zeta_2\tau_t^{10}(1-f_i^2)}{(f_i^2 - 1 - \tau_t^{10}\xi^{-2})},$$
$$h_i = -\lambda^* + (\lambda^* + 2\mu^*)(f_i^2 - \tau_c^{11}\xi^{-2}n_i'), \quad i = 1, 2.$$

Further in the absence of diffusion and thermal effects, i.e., if we take a=b=β_1=β_2= 0, in Eq. (6.1), we obtain the corresponding complex dispersion equation at the interface of viscous compressible fluid half-space and homogeneous isotropic elastic half-space as

$$d^* S_1 + \iota q(\mu^* S_2 - \omega S_3) + \omega S_4 = 0, \quad (6.3)$$

where

$$d^* = -\lambda^* + (\lambda^* + 2\mu^*)q^2, \qquad q^2 = 1 - c^2.$$

Eq. (6.3), by changing dimensionless quantities into physical quantities, is similar as obtained by Nayfeh and Nagy [17].

7 Amplitudes of displacements, temperature change and concentration

In this section, the amplitudes of displacement components, temperature change and concentration have been computed for stress free, thermal and concentration boundaries of homogeneous isotropic generalized thermoelastic diffusive half-space. Upon

using Eqs. (3.6) and (4.2a)-(4.2d), we obtain

$$u = \left\{\iota(e^{-\xi m_1 z} + Le^{-\xi m_2 z} + Me^{-\xi m_3 z}) + Nm_4 e^{-\xi m_4 z}\right\}\xi A_1 e^{\iota\xi(x-ct)}, \tag{7.1a}$$

$$w = \left\{ -(m_1 e^{-\xi m_1 z} + Lm_2 e^{-\xi m_2 z} + Mm_3 e^{-\xi m_3 z}) + \iota N e^{-\xi m_4 z}\right\}\xi A_1 e^{\iota\xi(x-ct)}, \tag{7.1b}$$

$$T = \left\{n_1 e^{-\xi m_1 z} + Ln_2 e^{-\xi m_2 z} + Mn_3 e^{-\xi m_3 z} + Nn_4 e^{-\xi m_4 z}\right\}A_1 e^{\iota\xi(x-ct)}, \tag{7.1c}$$

$$C = \left\{k_1 e^{-\xi m_1 z} + Lk_2 e^{-\xi m_2 z} + Mk_3 e^{-\xi m_3 z} + Nk_4 e^{-\xi m_4 z}\right\}A_1 e^{\iota\xi(x-ct)}, \tag{7.1d}$$

where

$$L = -\frac{m_1(n_1k_3 - n_3k_1)}{m_2(n_2k_3 - n_3k_2)}, \qquad M = \frac{m_1(n_1k_2 - n_2k_1)}{m_3(n_2k_3 - n_3k_2)},$$
$$N = -\frac{2\iota m_1\left[(n_2k_3 - n_3k_2) - (n_1k_3 - n_3k_1) + (n_1k_2 - n_2k_1)\right]}{(m_4^2+1)(n_2k_3 - n_3k_2)}.$$

8 Numerical results and discussion

With the view of illustrating theoretical results obtained in the preceding sections and compare these in the context of various theories of thermoelastic diffusion. We now represent some numerical results for copper material (thermoelastic diffusive solid), the physical data for which is given below:

$\lambda = 7.76 \times 10^{10} kgm^{-1}s^{-2}$, $\mu = 3.86 \times 10^{10} kgm^{-1}s^{-2}$, $T_0 = 0.293 \times 10^3 K$,
$C_E = .3831 \times 10^3 Jkg^{-1}K^{-1}$, $\alpha_t = 1.78 \times 10^{-5} K^{-1}$, $\alpha_c = 1.98 \times 10^{-4} m^3 kg^{-1}$,
$a = 1.2 \times 10^4 m^2 s^{-2} K^{-1}$, $b = 9 \times 10^5 kg^{-1} m^5 s^{-2}$, $D = 0.85 \times 10^{-8} kgsm^{-3}$,
$\rho = 8.954 \times 10^3 kgm^{-3}$, $K = 0.383 \times 10^3 Wm^{-1}K^{-1}$, $\tau_0 = 0.2s$,
$\tau_1 = 0.9s$, $\tau^0 = 0.3s$, $\tau^1 = 0.4s$.

The values of physical constants for different viscous fluids (Seawater and Gasoline) are given as:

Seawater

$$\rho_f = 1.025 \times 10^3 kgm^{-3}, \quad \eta = 1.07 \times 10^{-7} kgm^{-1}s^{-1}, \quad K_f = 2.33 \times 10^9 Nm^{-2}.$$

Gasoline

$$\rho_f = 0.68 \times 10^3 kgm^{-3}, \quad \eta = 0.292 \times 10^{-7} kgm^{-1}s^{-1}, \quad K_f = 0.958 \times 10^9 Nm^{-2}.$$

Figs. 2 and 3 show a flow chart to compute the phase velocity and attenuation coefficient of leaky rayleigh waves by using the Muller's method. Eq. (6.1) is a complex polynomial equation in two unknowns ξ and ω. For a given value of ω, Eq. (6.1) can be written as $F(\xi)$=0. Muller's method is used to find an estimated root of $F(\xi)$=0. The algorithm of Muller's method to find phase velocity and attenuation coefficient is as follows:

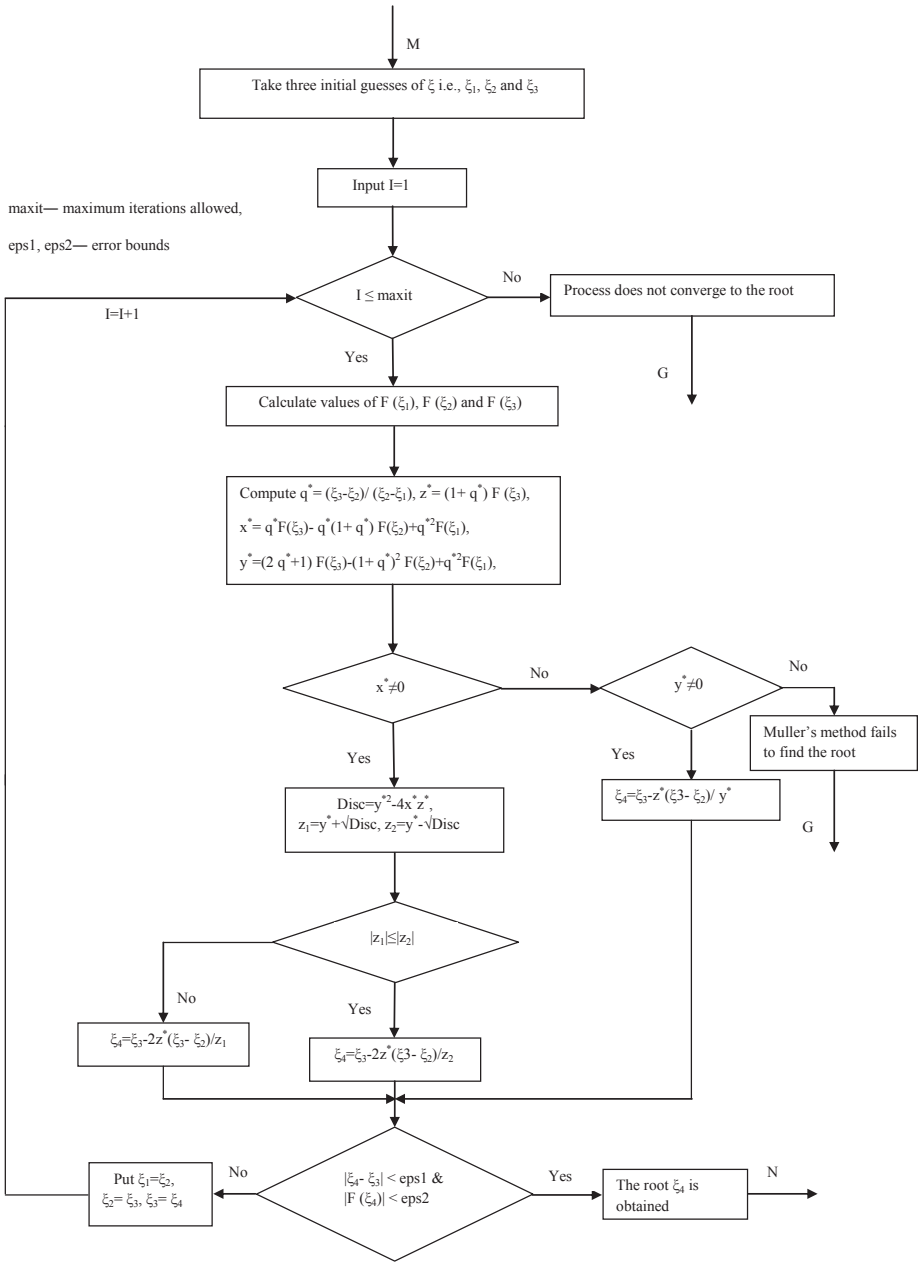

Figure 2: Flow chart to find an estimate root of $F(\xi) = 0$.

1. Decide initially three approximations say ξ_1, ξ_2 and ξ_3 of the root, number of iterations (maxit) and two error bounds (eps1 and eps2).
2. Put $I = 1$.
3. If $I \leq$ maxit, then compute $F(\xi_1)$, $F(\xi_2)$ and $F(\xi_3)$, otherwise write "Process fails to converge the root", and go to Step 10.

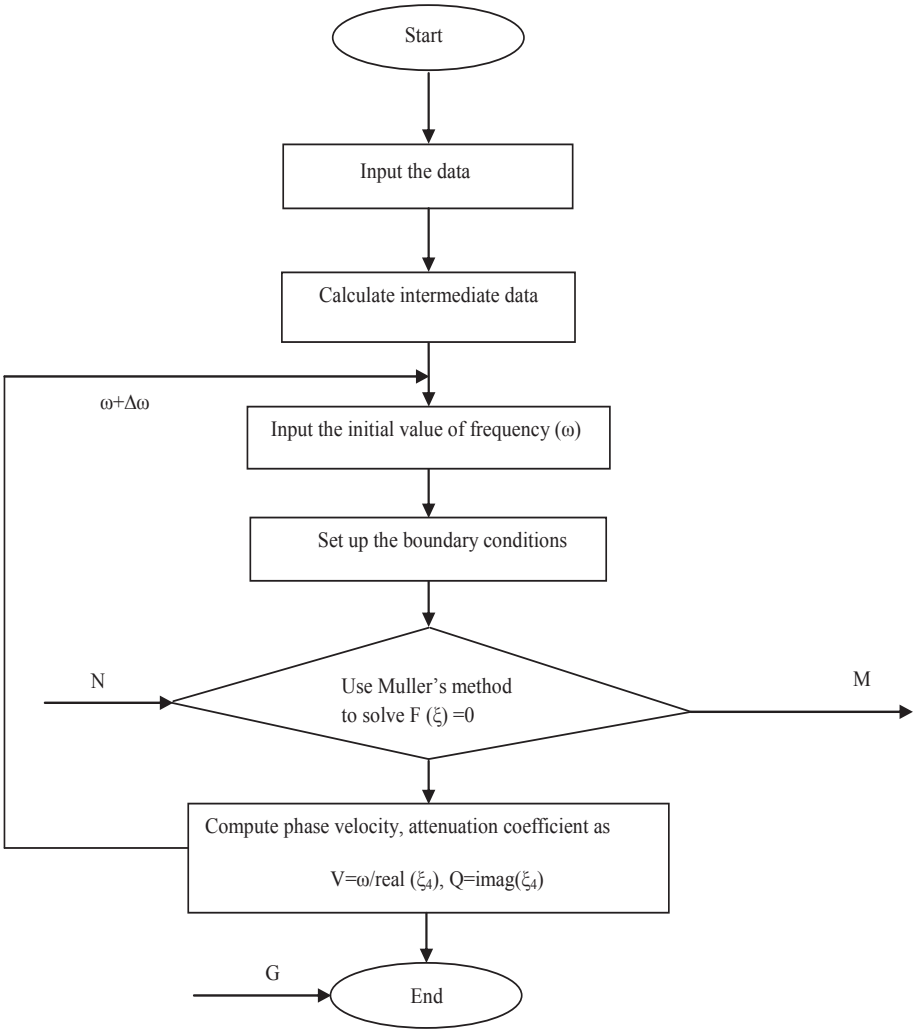

Figure 3: Flow chart for calculating phase velocity, attenuation coefficient.

4. Compute q^*, x^*, y^* and z^* by the following relations:

$$q^* = \frac{\xi_3 - \xi_2}{\xi_2 - \xi_1}, \qquad x^* = q^* F(\xi_3) - q^*(1+q^*)F(\xi_2) + q^{*2}F(\xi_1),$$
$$y^* = (2q^*+1)F(\xi_3) - (1+q^*)^2 F(\xi_2) + q^{*2}F(\xi_1),$$
$$z^* = (1+q^*)F(\xi_3).$$

5. If $x^* \neq 0$, then calculate

$$\text{discriminant (disc)} = y^{*2} - 4x^*z^*,$$
$$z_1 = y^* + \sqrt{disc}, \quad \text{and} \quad z_2 = y^* - \sqrt{disc}.$$

6. If $|z_1| \leq |z_2|$, then compute

$$\xi_4 = \xi_3 - \frac{2z^*(\xi_3 - \xi_2)}{z_2},$$

otherwise compute

$$\xi_4 = \xi_3 - \frac{2z^*(\xi_3 - \xi_2)}{z_1}.$$

7. If $|\xi_4 - \xi_3|$<eps1, and if $|F(\xi_4)|$<eps2, then root ξ_4 is obtained and go to Step 9. Otherwise put ξ_1=ξ_2, ξ_2=ξ_3 and ξ_3=ξ_4, I=$I+1$ and go to Step 3.
8. Otherwise if x^*=0, then check whether $y^* \neq 0$, or y^*=0. If $y^* \neq 0$, then calculate

$$\xi_4 = \xi_3 - \frac{z^*(\xi_3 - \xi_2)}{y^*},$$

and go to Step 7. If not, write "Muller's method fails to find the root" and go to Step 10.
9. Compute phase velocity (V) and attenuation coefficient (Q) as

$$V = \frac{\omega}{real(\xi_4)}, \qquad Q = imag(\xi_4).$$

10. Stop the process.

Similarly, we can find an estimated root of dispersion equation corresponding to stress free, thermal and concentration boundaries of homogeneous isotropic generalized thermoelastic diffusive half-space. After that, the magnitude of heat flux vector can obtained as:

$$|\vec{q}| = \sqrt{q_1^2 + q_3^2} = \frac{K}{\tau_t^0}\sqrt{\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2}, \tag{8.1}$$

where τ_t^0=1 for CT and G-L models, and τ_t^0=$1 + \tau_0 \partial/\partial t$ for L-S model. In the similar way, the magnitude of mass diffusion flux vector is obtained as:

$$|\vec{\eta}| = \sqrt{\eta_1^2 + \eta_3^2} = \frac{D}{\tau_f^0}\sqrt{\left(\frac{\partial P}{\partial x}\right)^2 + \left(\frac{\partial P}{\partial z}\right)^2}, \tag{8.2}$$

where τ_f^0=1 for CT and G-L models, and

$$\tau_f^0 = 1 + \tau^0 \frac{\partial}{\partial t},$$

for L-S model. The phase velocity and the attenuation coefficient of wave propagation

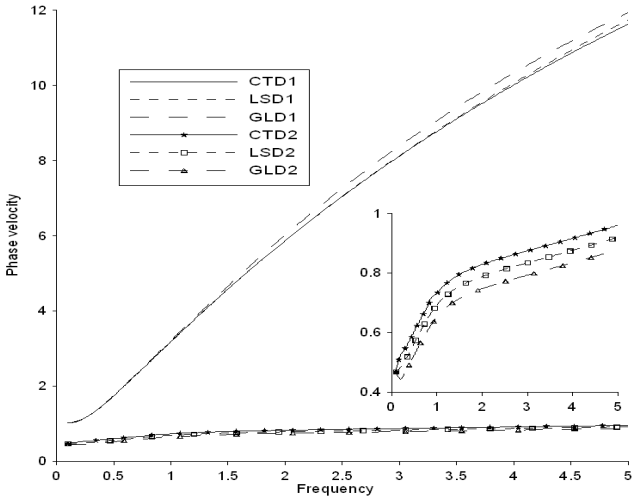

Figure 4: Variations of phase velocity w.r.t. frequency.

in the context of CT, L-S and G-L theories of thermoelastic diffusion have been represented graphically in Figs. 4-9 for various the values of frequency. The viscosity effect on phase velocity and attenuation coefficient in the context of CT, L-S and G-L theories of thermoelastic diffusion have been shown in Figs. 4 and 5 respectively. Similarly, the diffusion effect on phase velocity and attenuation coefficient in the context of CT, L-S and G-L theories of thermoelastic diffusion have been shown in Figs. 6 and 7 respectively. The magnitude of heat and mass diffusion flux vectors in the context of CT, L-S and G-L theories of thermoelastic diffusion have also been represented in Figs. 8 and 9 respectively. In Figs. 4-7, the solid, small dash and big dash lines correspond to CT, L-S and G-L theories of thermoelastic diffusion for Seawater and these are represented by CTD1, LSD1 and GLD1 respectively, whereas for Gasoline, in Figs. 4 and 5,

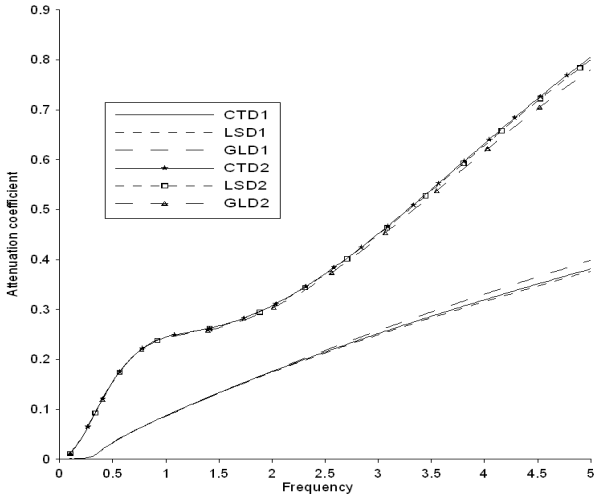

Figure 5: Variations of attenuation coefficient w.r.t. frequency.

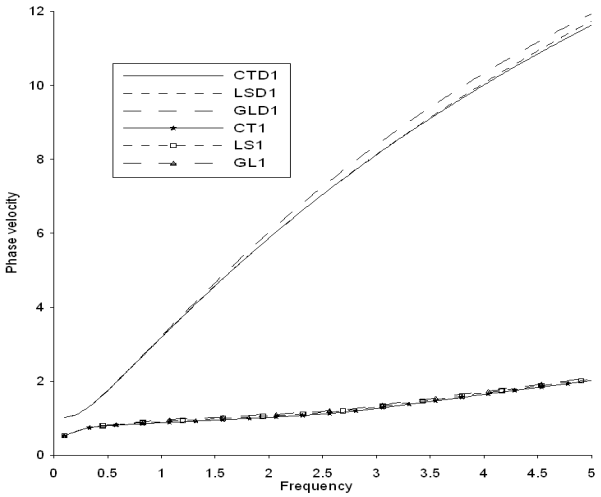

Figure 6: Variations of phase velocity w.r.t. frequency.

the star, square and triangle symbols on these lines correspond to CT, L-S and G-L theories of thermoelastic diffusion and these are represented by CTD2, LSD2 and GLD2 respectively. In Fig. 4, small figure is made to show the effect of relaxation times for Gasoline. In Figs. 6 and 7, the star, square and triangle symbols on solid, small dash and big dash lines, respectively, correspond to CT, L-S and G-L theories of thermoelasticity for Seawater and these are represented by CT1, LS1 and GL1 respectively. In Figs. 8 and 9, the solid, small dash and big dash lines correspond to CT, L-S and G-L theories of thermoelastic diffusion and these are represented by CTD, LSD and GLD respectively.

8.1 Phase velocity

From Fig. 4, it is noticed that corresponding to Seawater and Gasoline, the values of phase velocity increase with the increase in the values of frequency for all theories of thermoelastic diffusion. The reason is that as the value of frequency increases, the value of estimate root, that is, complex wave number increases, but the increase in the value of complex wave number is smaller as compared to the increase in the value of frequency. Therefore value of phase velocity (=frequency/real (wave number)) increases. The phase velocity for GLD1 remains higher than that of CTD1 and LSD1. On the other hand, the phase velocity for CTD2 remains lower than that of LSD2 and GLD2. Thus for Seawater, the effect of relaxation times increase the values of phase velocity, whereas for Gasoline, the effect of relaxation times decreases the values of phase velocity. On comparing the values of phase velocity for Seawater and Gasoline, we depict that the values of phase velocity are more corresponding to Seawater as compared to Gasoline.

It is evident from Fig. 6 that for Sewater, the values of phase velocity increase corresponding to three theories of thermoelasticity. The values for the case of CT1

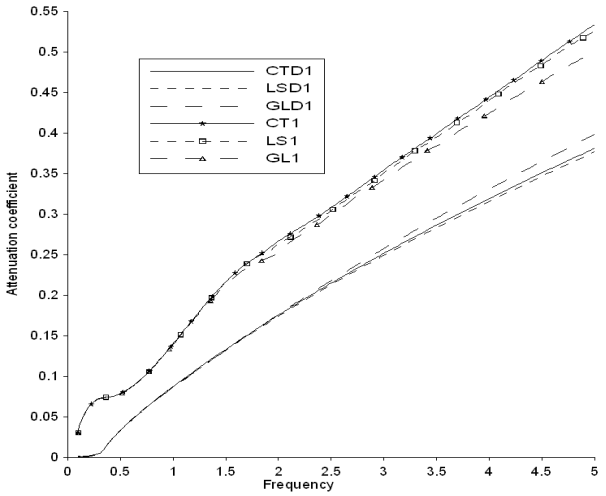

Figure 7: Variations of attenuation coefficient w.r.t. frequency.

are lower as compared to the cases of LS1 and GL1. On comparing the three theories of thermoelastic diffusion and the three theories of thermoelasticity, we find that for Seawater, the diffusion effect increases the values of phase velocity.

8.2 Attenuation coefficient

It is evident from Fig. 5 that the values of attenuation coefficient corresponding to three theories of thermoelastic diffusion increase with the variation in frequency for both viscous fluids Seawater and Gasoline. As already mentioned, the value of complex wave number increases with the increase in value of frequency, therefore the value of attenuation coefficient (=imaginary (wave number)) increases. On comparing the

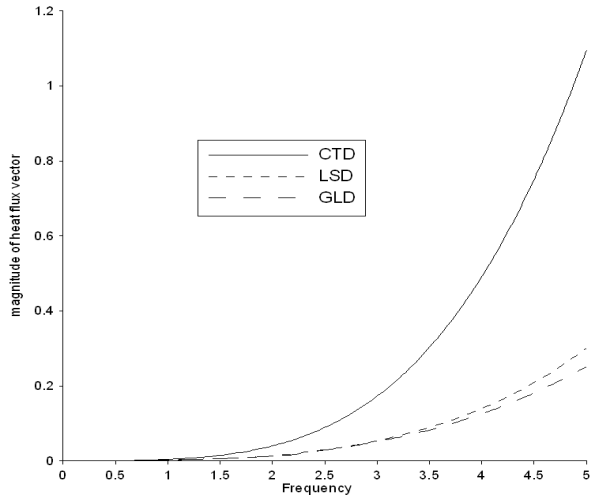

Figure 8: Variations of magnitude of heat flux vector w.r.t. frequency.

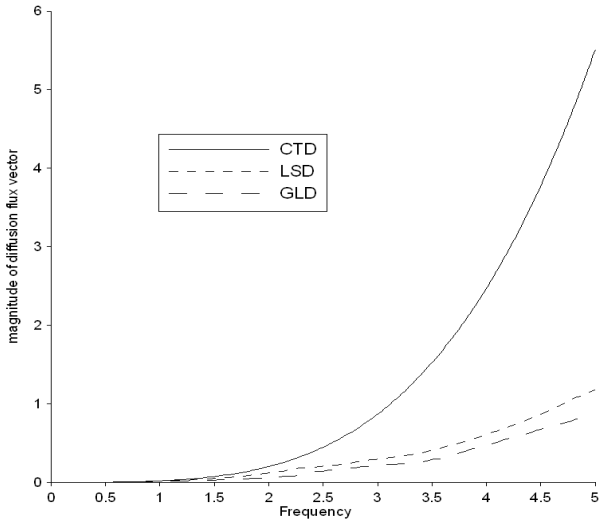

Figure 9: Variations of magnitude of diffusion flux vector w.r.t. frequency.

results for CTD1, LSD1 and GLD1, we find that the values of attenuation coefficient corresponding to GLD1 are more in comparison to CTD1 and LSD1, whereas the values are higher for CTD2 as compared to other two theories. On comparing the cases of Gasoline and Seawater, we depict that the values corresponding to Gasoline remain higher than that of Seawater.

It is noticed from Fig. 7 that for Sewater, the values of attenuation coefficient increase corresponding to three theories of thermoelasticity. The values for the case of GL1 are lower as compared to the cases of CT1 and LS1. We note that the values of attenuation coefficient corresponding to three theories of thermoelasticity are higher in comparison to the three theories of thermoelastic diffusion.

8.3 Heat flux

Fig. 8 clearly indicates that the values of magnitude of heat flux vector increase corresponding to three theories of thermoelastic diffusion. The values of magnitude of heat flux vector for CTD are more than that of LSD and GLD. Thus, we find that the effect of relaxation times decreases the values of magnitude of heat flux vector.

8.4 Mass diffusion flux

It is observed from Fig. 9 that the values of magnitude of mass diffusion flux vector increase corresponding to CTD, LSD and GLD. The values of magnitude of mass diffusion flux vector increase higher for CTD as compared to LSD and GLD. So we notice that if we neglect the effect of relaxation times, the values increase. In Figs. 8 and 9, the magnitude of heat flux and mass diffusion flux vectors are demagnified by 10^{-5} times to show the effect of relaxation times.

9 Conclusions

In the present paper, the propagation of leaky Rayleigh waves in a viscous compressible fluid half-space overlying a homogeneous isotropic, thermoelastic diffusive half-space in the context of generalized theories of thermoelastic diffusion has been studied. The phase velocity and attenuation coefficient of leaky Rayleigh waves have been computed from the complex dispersion equation by using the Muller's method. The amplitudes of displacements, temperature change and concentration have been obtained. The viscosity and diffusion effects on phase velocity and attenuation coefficient of leaky Rayleigh waves motion for different theories of thermoelastic diffusion have been depicted. In addition, the magnitude of heat and mass diffusion flux vectors for different theories of thermoelastic diffusion have also been computed.

It is observed that for Seawater and Gasoline, the values of phase velocity and attenuation coefficient increase with the increase in the value of frequency corresponding to three theories of thermoelastic diffusion. Corresponding to Seawater, the values of phase velocity are higher in comparison to Gasoline. But the values of attenuation coefficient are more corresponding to Gasoline as compared to Seawater. If we neglect the diffusion effect, the values of phase velocity decrease, whereas the values of attenuation coefficient increase. The values of magnitude of heat flux and mass diffusion flux vectors increase in all theories of thermoelastic diffusion. If we neglect the effect of relaxation times, the values of magnitude of heat flux and mass diffusion flux vectors decrease.

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