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Superconvergence of Mixed Methods for Optimal Control Problems Governed by Parabolic Equations

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Abstract. In this paper, we investigate the superconvergence results for optimal control problems governed by parabolic equations with semidiscrete mixed finite element approximation. We use the lowest order mixed finite element spaces to discrete the state and costate variables while use piecewise constant function to discrete the control variable. Superconvergence estimates for both the state variable and its gradient variable are obtained.

AMS subject classifications: 65L10, 65L12

Key words: Optimal control, mixed finite element, superconvergence, parabolic equations.

1 Introduction

Optimal control problems [33] have been extensively utilized in many aspects of the modern life such as social, economic, scientific and engineering numerical simulation. Due to the wide application of these problems, it must be solved successfully with efficient numerical methods. Among these numerical methods, finite element discretization of the state equation is widely applied though other methods are also used. There have been extensive studies in convergence of finite element approximation of optimal control problems, see, for example [1, 2, 10–13, 15, 24, 29, 30, 37, 38, 45]. A systematic introduction of finite element method for PDEs and optimal control can be found in, for example, [16, 28, 42], and [44].

Many researchers have made a lot of works on some topics of finite element methods for optimal control problems. In particular, for optimal control problem governed by linear elliptic state equations, there are two early papers on the numerical approximation for linear-quadratic control-constrained problems by Falk [23] and Geveci [26].

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More recently, Arnautu and Neittaanmäki [3] contributed further error estimates to this class of problems. Moreover, we refer to Casas [7] who proves convergence results for optimal control problems governed by linear elliptic equations with control in the coefficient. Most recently, C. Meyer and A. Rösch have studied the superconvergence property for linear-quadratic optimal control problem in [41], W. B. Liu and N. N. Yan [35] and [34] have derived a posteriori error estimates for finite element approximation of convex optimal control problems and boundary control problems respectively.

For optimal control problem governed by linear parabolic state equations, a priori error estimates of finite element approximation were studied in, for example [30] and [32]. A posteriori error estimates for this problem were discussed by W. B. Liu and N. N. Yan [36]. Notice that all the above works are mainly focused on standard finite element methods.

But the mixed finite element method is much more important for a certain class of problems which contains the gradient of the state variable in the objective functional. Thus the accuracy of gradient is of great importance in numerical approximation of the state equations. When it comes to these problems, it is advantageous to apply mixed finite element methods with which both the state variable and its gradient variable can be approximated with the same accuracy. Although mixed finite element methods has been extensively used in engineering numerical simulations, it has not been fully utilized in computational optimal control problems yet. Particularly, there has not been much work on theoretical analysis of mixed finite element approximation for parabolic optimal control problem in the literature although there are some papers about the mixed finite element methods for parabolic equation, for example, see [8, 9, 14, 21] and [25]. In [8] and [21], the authors derived superconvergence for the mixed finite element approximations to parabolic problems.

Superconvergence results are important from an application point of view since, under reasonable assumptions on the grid and with additional smoothness of the solution, they provide higher accuracy. There has been much work on superconvergence of elliptic problems for the rectangular or quadrilateral finite element partition by mixed methods, see [18–20], and [22]. But for optimal control problems governed by parabolic equations there exist no superconvergence results of mixed methods. In our priori work [46], we have established the L^2-error estimates for this optimal control problems. We can see that the L^2-error both for the control and the state is of $\mathcal{O}(h)$.

In this paper, we will prove superconvergence results on rectangular domain for the optimal control problems governed by parabolic equation using mixed methods. More precisely, we shall prove that the finite element solution is superclose to a certain projection of the exact solution. The paper is organized as follows: in Section 2, we construct the discrete scheme of this problem by using mixed finite element methods and give its equivalent optimality conditions. Then, we present some preliminary results in Section 3. The main theorems on superconvergence of this paper are formulated in Section 4. Finally, in Section 5, we make a conclusion and state some future