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Flow of Newtonian Fluid in Non-Uniform Tubes with Application to Renal Flow: A Numerical Study

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Abstract. In this paper, a numerical method employing a finite difference technique is used for an investigation of viscous, incompressible fluid flow in a tube with absorbing wall and slowly varying cross-section. The effect of fluid absorption through permeable wall is accounted by prescribing flux as a function of axial distance. The method is not restricted by the parameters in the problem such as wave number, permeability parameter, amplitude ratio and Reynolds number. The effects of these parameters on the radial velocity and mean pressure drop is studied and the results are presented graphically. Comparison is also made between the results obtained by perturbation method of solution and present approach.

AMS subject classifications: 76Z05, 92C10

Key words: Non-uniform tube, renal flow, Takabatake finite difference scheme.

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1 Introduction

The process of reabsorption plays a major role during urine formation in kidneys as 98 percent of glomerular filtrate gets reabsorbed during its passage through renal tubules. Kidneys excrete most of the end products of body metabolism and they control concentrations of most of the constituents of body fluids. The basic functional unit of kidney is nephron. Each kidney contains over a million tiny units (of nephrons), all similar in structure and function. Each nephron functions independently and in most instances it is sufficient to study the function of nephron to understand the mechanism of kidney in terms of mathematical models. In nephrons, the portion after the Bowman's capsule is called proximal convoluted tubule, which is narrower than rest of the tube and non-uniform in nature. It is the place where most of useful substances, like water, glucose and electrolytes are reabsorbed back into the plasma and unwanted

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substances pass into urine. Thus it is of interest to study the flow in proximal tubule using mathematical models.

Study of viscous fluid flow in channels of varying cross section with permeable wall is significant because of its applications to both physiological and engineering flow problems. The flow of fluid in a renal tubule has been studied by various authors. Macey [1] formulated the problem as the flow of an incompressible viscous fluid through a circular tube with linear rate of reabsorption at the wall. Whereas, Kelman [2] found that the bulk flow in the proximal tubule decays exponentially with the axial distance. Then, Macey [3] used this condition to solve the equations of motion and mentioned that the longitudinal velocity profile is parabolic and the drop in mean pressure is proportional to the mean axial flow. Marshall and Trowbridge [4] and Palatt et al. [5] used physical conditions existing at the rigid permeable tube instead of prescribing the flux at the wall as a function of axial distance.

The representation of a proximal tubule as a uniform tube with constant wall permeability is obviously an idealization. Radhakrishnamacharya et al. [6] considered a non-uniform geometry to model renal tubule while the previous studies considered it uniform. They made an attempt to understand the flow through the renal tubule by studying the hydrodynamical aspects of an incompressible viscous fluid in a circular tube of exponentially varying cross-section with reabsorption at the wall. Following similar approach, Chandra and Prasad [7] analyzed fluid flow in rigid tube of slowly varying cross-section by considering different geometries. Also they investigated the problem by considering fluid exchange across the permeable wall governed by Starling's hypothesis. Chaturani and Ranganatha [8] studied fluid flow through a diverging/converging tube with variable wall permeability. They obtained approximate analytical solution for the case that the flux at the wall depends on wall permeability and transboundary pressure drop. Recently, Muthu and Tesfahun [9] have studied the fluid flow in nonuniform rigid wavy channel of varying cross section and presented the effects of slope parameter, reabsorption coefficient on the transverse velocity and mean pressure drop.

In all the above studies, the method used to solve the governing equations of the fluid motion is perturbation method of solution by taking small nonuniform tube parameter/curvature parameter. As per the knowledge of the authors there is no numerical study of the above problems reported in the literature.

Hence, in this paper, the Navier-Stokes equations governing the flow of an incompressible viscous fluid through a wavy (rigid diverging/converging tube of varying cross-section) non-uniform permeable tube are solved numerically by using the finite difference technique related to the method of Takabatake-Ayukawa [10]. The effects of wave number (δ), reabsorption coefficient (α), amplitude ratio (ϵ) and Reynolds number on the transverse velocity, stream function and mean pressure drop are studied without restrictions on the parameters of the problem, in principle. Further, we compared the results found by the current approach with that of perturbation method of solution.

The boundary of the tube wall vary with x. It is taken as