

# A Hybrid Method for Dynamic Mesh Generation Based on Radial Basis Functions and Delaunay Graph Mapping

Li Ding, Tongqing Guo and Zhiliang Lu\*

*Department of Aerodynamics, Nanjing University of Aeronautics and Astronautics,  
29 Yudao Street, Nanjing 210016, China*

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**Abstract.** Aiming at complex configuration and large deformation, an efficient hybrid method for dynamic mesh generation is presented in this paper, which is based on Radial Basis Functions (RBFs) and Delaunay graph mapping. Based on the computational mesh, a set of very coarse grid named as background grid is generated firstly, and then the computational mesh can be located at the background grid by Delaunay graph mapping technique. After that, the RBFs method is applied to deform the background grid by choosing partial mesh points on the boundary as the control points. Finally, Delaunay graph mapping method is used to relocate the computational mesh by employing area or volume weight coefficients. By applying different dynamic mesh methods to a moving NACA0012 airfoil, it can be found that the RBFs-Delaunay graph mapping hybrid method is as accurate as RBFs and is as efficient as Delaunay graph mapping technique. Numerical results show that the dynamic meshes for all test cases including one two-dimensional (2D) and two three-dimensional (3D) problems with different complexities, can be generated in an accurate and efficient manner by using the present hybrid method.

**AMS subject classifications:** 65Z05

**Key words:** Dynamic mesh method, hybrid method, background grid, radial basis functions, delaunay graph mapping.

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## 1 Introduction

Dynamic mesh is critical for many Computational Fluid Dynamics (CFD) problems such as flutter, static aeroelastics, optimization design of high lift system, and so on. Grid regeneration at every time step is often encountered for a dynamic system. However,

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\*Corresponding author.

*Email:* luzl@nuaa.edu.cn (Z. L. Lu)

it is not only extremely time-consuming but also very inconvenient. Therefore, efficient methods are of great importance to generate dynamic mesh with high mesh quality.

With the development of CFD, more and more dynamic mesh generation methods have been proposed, such as Transfinite Interpolation (TFI) [1–4], spring analogy [5–7], inverse distance weighted (IDW) [8, 9], elasticity method [10–12], radial basis functions (RBFs) [13–15], Delaunay graph mapping [16], and so on. Each method has its own advantages and disadvantages. Basically, RBF is time consuming but may lead to better quality mesh in relation to skewness. Delaunay mapping method is efficient but the skewness of the deformed mesh is more difficult to control. When the deformation is large or the geometry shape is complex, the mesh quality cannot be kept.

Recently, more and more hybrid methods have been developed. Ding [17] applied RBFs and TFI for complex multi-block structured grid. Spekrijse [18] developed a grid deformation algorithm for multi-block structured grid, which is based on volume spline interpolation and TFI. Zhou [19] combined the spring analogy with Delaunay graph mapping to generate the dynamic mesh with large deformation. Although various dynamic mesh generation methods have been continuously improved and have gotten wide applications, dynamic mesh generation is still very challenging in the cases of complex configuration, large grid number and large deformation.

In this paper, a hybrid method for dynamic mesh generation is developed by combining RBFs with Delaunay graph mapping. In order to improve the Delaunay graph mapping to deal with complex configuration and large deformation well, the background grid used in Delaunay graph mapping, which only consists of mesh points on the boundary, is replaced by a set of fairly coarse unstructured background grid. The background grid contains moderate mesh points in the interior flow field and maintains the same mesh distributions as the given computational mesh on the wall surface. Then RBFs method is utilized to deform the background grid by choosing partial points on the boundary as the control points, and Delaunay graph mapping method is used to deform the computational mesh by employing area or volume weight coefficients. Different methods are used for a moving NACA0012 airfoil, and the comparison indicates that RBFs-Delaunay graph mapping hybrid method is as accurate as RBFs and is as efficient as Delaunay graph mapping. The present hybrid method is applied to one 2D and two 3D dynamic mesh problems with different complexities, and the numerical results demonstrate that the dynamic meshes for all test cases can be generated in an accurate and efficient manner.

## 2 RBFs and Delaunay graph mapping

### 2.1 RBFs

RBFs [20, 21] use the interpolation function  $f$  to describe the displacement in the whole physical space, and  $f$  can be approximated by a weighted sum of the basis functions