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A Two-Level Method for Pressure Projection Stabilized P_1 Nonconforming Approximation of the Semi-Linear Elliptic Equations

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Abstract. In this paper, we study a new stabilized method based on the local pressure projection to solve the semi-linear elliptic equation. The proposed scheme combines nonconforming finite element pairs NCP_1-P_1 triangle element and two-level method, which has a number of attractive computational properties: parameter-free, avoiding higher-order derivatives or edge-based data structures, but have more favorable stability and less support sets. Stability analysis and error estimates have been done. Finally, numerical experiments to check estimates are presented.

AMS subject classifications: 35Q30, 74S05, 65N30

Key words: Semi-linear elliptic equations, two-level method, nonconforming finite element method, stabilized method.

1 Introduction

Let Ω be a bounded domain in R^2 with a Lipschitz boundary $\partial\Omega$. In this paper, we will consider the following semi-linear elliptic equations:

$$\begin{cases} -\Delta p - f(p) = g(x) & \text{in } \Omega, \\ p = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $p(x)$ is an unknown variable and $g(\mathbf{x}) \in L^2(\Omega)$.

A direct way to solve this model is Galerkin finite element method. However, the regularity of solution is higher if this approach is used. This leads to some difficulties in practical application. An alternative formulation has been used to reduce the need of

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regularity such as mixed finite element method [1]. The classic Galerkin finite element method of mixed formulation require the finite element pairs to satisfy the LBB conditions, but the continuous LBB conditions are not naturally inherited by most mixed finite element spaces. This limits the use of low-order finite element spaces, which do not satisfy LBB condition. Especially, the lowest equal-order pairs, such as $P1-P1$ (or Q_1-Q_1) pairs are of practical important in scientific computation, because they are computationally convenient and efficient in a parallel or multi-grid context [2]. Particular concern is the nonconforming finite element methods, which is even more attractive due to the simplicity and convenience. Compared with the conforming finite element methods, the nonconforming finite element not only have more favorable stability and less support sets, but can also relax the requirement of regularity [3, 4]. Hence, the nonconforming finite element seems more suitable to solve this type of problem.

To overcome the lack of LBB stability, a lot of stabilized techniques have been proposed. Many of these stabilized techniques depend on the stabilized parameter either explicitly or implicitly [5–10]. In practice, these parameter are still being determined by trial and error. Moreover, these is no satisfied answer to the stabilized parameter in all situations. Simultaneously, another class of stabilized method is proposed, which is not require specification of a stabilization parameter [11–14]. This stabilized strategy requires edge-based data structures and a subdivision of grid into patches. Recently, based on polynomial pressure, a new family of stabilized method is presented [15–17], which has some prominent features: parameter free, avoiding higher-order derivatives or edge-based data structures, and stabilization being completely local at the element level. Some of the above techniques have also been extended to transient incompressible flow problems [18, 19].

On the other hand, the numerical solution of a nonlinear system can be very time consuming, two-level methods are considered as an effective alternative method to solve this kind of problem. Two-level methods aim to obtain a discrete approximate solution of a nonlinear partial equation with less computional cost and to preserve the optimal order of convergence. The basic idea is to solve a complicated problem on the coarse grid, then solve a simple symmetric positive or linearized problem on fine grid. This method is first introduced by Xu [20–23] for nonsymmetric linear and linear elliptic problem. Later on, two-grid method is further investigated [24–27]. Due to the validity of two-grid methods in practical computation, this kind of method has been widely applied to the various of nonlinear problem, such as the semi-linear reaction-diffusion model [28, 29], and the semi-linear hyperbolic equation [30].

In this paper, we introduce the stabilized finite element scheme of the semi-linear elliptic equation based on the lowest equal-order pairs of nonconforming mixed finite element pairs (i.e., NCP_1-P_1). Moreover, two-level method is used to reduce the time consuming in the proposed scheme. The stabilized method is based on the local pressure projection [15]. This paper is organized as follows: in next section, an abstract functional setting for the semi-linear elliptic equations is presented, together with some basic notation. Then, in Section 3, the stabilized nonconforming mixed finite element scheme is