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Impact of Linear Operator on the Convergence of HAM Solution: a Modified Operator Approach

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Abstract. The linear operator plays an important role in the computational process of Homotopy Analysis Method (HAM). In HAM frame any kind of linear operator can be chosen to develop a solution. Hence, it is easy to introduce the modified/physical parameter dependent linear operators. The effective use of these operators has been judged through solving fluid flow problems. Modification in linear operators affects the solution and improves the computational efficiency of HAM for larger values of parameters. The convergence rate of the solution is rapid and several times higher resulting in lesser computational time.

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Key words: Optimal homotopy analysis method (OHAM), modified operator, rapid convergence, large physical parameters, stretching surface.

1 Introduction

Homotopy Analysis Method (HAM) is being used to obtain the solutions of highly nonlinear differential equations. Firstly this technique was introduced by Liao in his Ph.D thesis and developed it further by introducing a convergence control parameter in the deformation equation. His book [1] thoroughly describes the application of HAM. This technique has several advantages over the perturbation methods and other analytic techniques. For example; no small or large parameter restriction, freedom to choose the linear

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operator, existence of efficient scheme to examine the convergence of obtained solutions, uniform applicability to the ordinary and partial differential equations and successful application for solving the highly non-linear system of differential equations [2–11]. The applicability and use of this method over the years resulted in a great improvement therein. Marinca and Herisanu [12,13] first introduced a number of convergence control parameters and thereafter used the concept of optimization for obtaining the best values. Although this concept is rigorous theoretically yet it is less efficient computationally. Liao by taking into account computational efficiency, introduced three parameter based Optimal Homotopy Analysis Method (OHAM). In his recent paper [14] he, however suggested that the basic OHAM (i.e., which contains one convergence control parameter) is computationally more efficient than three parameters OHAM, finite parameters OHAM and infinite parameters OHAM. Recently Zhao et al. [15], introduced the modified HAM, orthonormal functions have been used to control the rapid growth of terms in higher order approximations. This approach reduced the standard computational time as compared to the standard HAM. Shehzad et al. [16] discussed the magnetohydrodynamic flow of casson fluid over a stretching sheet. They plotted the h-curve at $M=0.5$ and showed the convergence of solution for small value of M. The variation of velocity profile for $M=0$ to 2 was discussed and tables were drawn for maximum M equals to 1.6. Hayat et al. [17] discussed the flow and heat transfer phenomenon in magneto hydrodynamic fluid over a permeable stretching sheet in the presence of radiations. They examined the convergence of solution by assuming M and Pr equal to 1. Due to use of simple linear operators for momentum and heat transfer equation the convergent solutions for large values of M and Pr were difficult to obtain. The maximum values of M and Pr chosen in the article were 3 and 10 respectively. Nadeem and Saleem [18] discussed the flow of Nano fluid over a rotating cone in the presence of magnetic field. Their study reports the maximum value of M equals to 7 and Pr equals to 6. Ellahi and Riaz [19] modelled the third grade fluid flow with variable velocity and solved the governing equations by using the HAM. In their article the maximum value of M was reported to be 4. Xu and Liao [20] obtained the series solution for the unsteady MHD flow of non-Newtonian fluid due to impulsively stretching sheet and discussed the velocity variations for M equals to 0, 1 and 2. Liao [21] obtained the solutions for the Blasius flow problem in the larger domain of Prandtl number between 0 and 50. First he introduced an arbitrary parameter in the linear operator and later on chose its suitable value. The result obtained through this technique could only match up to three decimal points with numerical results. Although introduction of parameter in the linear operator was a good thought yet no precise way was suggested to choose the best value. One possible way could be the optimization of this parameter against the total squared residual error.

Although HAM is a strong analytic technique to solve highly nonlinear problems yet for large physical parameters it was not providing the best correlation with numerical solutions. Liao in his recent book [22] obtained the solutions by choosing different kind of basis set and linear operators.

In HAM frame any kind of linear operator and basis set can be chosen to develop a