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Superconvergence Analysis of Gradient Recovery Method for TM Model of Electromagnetic Scattering in the Cavity

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Abstract. In this paper, we consider the transform magnetic (TM) model of electromagnetic scattering in the cavity. By the Polynomial Preserving Recovery technique, we present superconvergence analysis for the vertex-edge-face type finite element. From the numerical example, we can see that the provided method is efficient and stable.

AMS subject classifications: 65N06, 65N22

Key words: Scattering, cavity, TM model, gradient recovery method, superconvergence.

1 Introduction

Electromagnetic scattering problems have been considered by many researchers and engineers because of the significant industrial and military applications, which include the design of cavity-backed conformal antennas, the characterization of radar cross-section (RCS) of vehicles with grooves and others [1–4].

Finite element method is one of the important ways to simulate electromagnetic scattering problems numerically [5, 8–12, 14]. Some extreme vital focuses should be payed more attentions including that: (1) One has to reduce the infinite problem domain to a bounded computational domain. Many methods have been designed for this purpose including absorbing boundary conditions (ABC) by X. Feng [15], perfectly matched layer (PML) Z. Chen [5–7], and transparent boundary conditions by G. Bao [2], A. W. Wood [3],

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K. Du [4] and many references therein. (2) Large wave number problem will occur, that is to say, the pollution error of the finite element solution has to be considered in the error analysis. The discontinuous Galerkin (DG) method or CIP-FEM by X. Feng and H. Wu [11–13] can deal with such a problem, and hybridizable discontinuous Galerkin (HDG) method provided by H. Chen, P. Lu and X. Xu [14] is successful, too.

Theoretically, scattering field in weak formulation belongs to the Sobolev space $H^1(\Omega)$ and its numerical approximation is very worse, even $\mathcal{O}(\epsilon)$, for any $\epsilon>0$ [16]. Practically, the above numerical ideas were realized, which based on an assumption that scattering fields were filled by materials with proper smooth relative permittivity parameters, seeing D. Arnold [17] and I. Babuska [18]. Therefore, one of facts is that the high convergent order can deduce the effect of large wave number.

Superconvergence analysis is one of excellent post-processing techniques with lower computational costs. The most widely used in practice is Zienkiewicz-Zhus Superconvergence Patch Recovery (SPR) method [19]. However, in [20] the authors showed that the SPR was not superconvergent for linear element under the uniform triangulation with the Chevron pattern grid. The Polynomial Preserving Recovery (PPR) which overcome this restriction was one of the least-squares-based procedures [21].

In this paper, we use PPR with the least-squares-based procedure to improve the convergence order for transverse magnetic (TM) model of Electromagnetic Scattering problem. This method is based on computing a local k order polynomial on a suitable patch associated with each mesh vertex via a discrete least-squares procedure. Then, the nodal gradient can be computed. Numerical example illustrates the effectiveness of this recovery method.

The paper is organized as follows. In Section 2, the TM model in a cavity is provided and the existence and uniqueness of variational problem are presented, too. In Section 3, we formulate vertex-edge-face type finite element and the gradient recovery procedure. In Section 4, superconvergence theory is established. In Section 5, numerical example tests our theories. In Section 6, conclusion in this paper is provided.

2 TM model in a cavity

TM model in a cavity can be formulated by [2,3,22]

$$\begin{cases} \Delta u+k_0^2\varepsilon_r u=f & \text{in } \Omega, \\ u=0 & \text{on } S, \\ \dfrac{\partial u}{\partial \mathbf{n}}=T(u)+g & \text{on } \Gamma, \end{cases} \tag{2.1}$$

where $g(x)=-2i\beta e^{i\alpha x}$, and

$$T(u)=\frac{ik_0}{2}\int_0^a \frac{1}{|x-x'|}H_1^{(1)}(k_0|x-x'|)u(x')dx' \tag{2.2}$$