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1D Exact Elastic-Perfectly Plastic Solid Riemann Solver and Its Multi-Material Application

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Abstract. The equation of state (EOS) plays a crucial role in hyperbolic conservation laws for the compressible fluid. Whereas, the solid constitutive model with elastic-plastic phase transition makes the analysis of the solid Riemann problem more difficult. In this paper, one-dimensional elastic-perfectly plastic solid Riemann problem is investigated and its exact Riemann solver is proposed. Different from previous works treating the elastic and plastic phases integrally, we resolve the elastic wave and plastic wave separately to understand the complicate nonlinear waves within the solid and then assemble them together to construct the exact Riemann solver for the elastic-perfectly plastic solid. After that, the exact solid Riemann solver is associated with the fluid Riemann solver to decouple the fluid-solid multi-material interaction. Numerical tests, including gas-solid, water-solid high-speed impact problems are simulated by utilizing the modified ghost fluid method (MGFM).

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Key words: Riemann problem, elastic-plastic solid, ghost fluid method, multi-material flows.

1 Introduction

In recent decades, exact and approximate Riemann solvers have been carefully designed to calculate the Riemann problem and provide the numerical fluxes across the interfaces of computing cells. Godunov firstly introduced a conservative method which defined the numerical fluxes by solving local Riemann problems to settle the non-linear systems of conservation laws [1]. In the original Godunov method, local Riemann problems are solved accurately. As the iterative procedure of the exact Riemann solver is rather time-consuming, alternatively, approximate and non-iterative Riemann solvers are developed to evaluate the numerical fluxes approximately. On one hand, approximate

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state-based Riemann solvers such as double-shock Riemann solver, double-rarefaction Riemann solver, and their hybrid versions [2] are employed to define the fluxes indirectly. On the other hand, approximate flux-based Riemann solvers, for instance, HLL-type Riemann solvers [3–6], Roe Riemann solver [7] and Osher Riemann solver [8, 9], estimate the fluxes directly. These two classes of approximate Riemann solvers have been widely adopted in hydrodynamics codes nowadays.

However, in this work, we study the exact Riemann solvers for the elastic-perfectly plastic solid and its multi-material interfaces. The reason is twofold. Firstly, as complex nonlinear waves arise in the elastic-plastic solid, the intensive study of the exact solution can provide a more reliable result and an initial foothold for the further more efficient approximate solution theoretically. Secondly, the exact Riemann solver is intended to treat the multi-material interaction by the MGFM approach [10–12] instead of developing a Godunov-type scheme. Since MGFM is a multi-material method which needs to solve only one local Riemann problem at the multi-material interface during each time step, the time-consuming effect is limited.

Actually, for purpose of solving the solid Riemann problem, a number of exact or approximate Riemann solvers based on the specific solid assumptions have been proposed. Regarding to the elastic solid, Kaboudian and Khoo [13] analysed the elastic solid Riemann problem in the Lagrangian framework and put forward a Riemann solver according to the characteristic theory. As for the nonlinear elasticity, multiple approaches have been implemented. LeFloch [14] only utilized shock waves to approximate the original Riemann problem structure. Garaizar [15] demonstrated an exact Riemann problem solution of isotropic hyperelastic material theoretically without performance demonstration by numerical results. Miller [16] presented a general iterative Riemann solver and applied it to the hyperelastic material. The great discrepancy between the exact and approximate solutions indicates that the exact solution is a necessity for the complex solid flow. Barton et al. [17] achieved high-order spatial accuracy by the monotonicity preserving weighted essentially non-oscillatory (MPWENO) reconstruction and the local Riemann problem characteristic decomposition. HLLD Riemann solver was implemented by López Ortega et al. [18] and then was extended to the multi-material circumstance. Considering the elastic-plastic solid, Trangenstein and Pember [19] illustrated an analytical solution to the Riemann problem for Antman-Szymczak model which described the elastic-plastic material. Lin [20] raised an iterative procedure to calculate the Riemann problem approximately by linearizing the Riemann invariants. Whereas Wang et al. [21] adopted the fully conservative Eulerian formulation for elasto-plasticity proposed in [22] and employed an approximate Riemann solver via right eigenvectors decomposition based on constant coefficient systems. Miller and Colella [23] also took the similar approach to construct a high-order Godunov method for the elastic-plastic flow in solid solved in nonconservative form depending on the work of Trangenstein and Colella [24].

A large part of the methods illustrated above introduce the deformation tensor and formulate a hyperbolic conservative system with variables like inverse deformation gradient, mass, momentum, energy, even work-hardening, plastic strain tensor, etc. Consid-