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Solution of Boundary Value Problems Using Dual Reciprocity Boundary Element Method

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Abstract. In this work we utilize the boundary integral equation and the Dual Reciprocity Boundary Element Method (DRBEM) for the solution of the steady state convection-diffusion-reaction equations with variable convective coefficients in two-dimension. The DRBEM is a numerical method to transform the domain integrals into the boundary only integrals by using the fundamental solution of Helmholtz equation. Some examples are calculated to confirm the accuracy of the approach. The results obtained by the analytic solutions are in good agreement with ones provided by the DRBEM technique.

AMS subject classifications: 65M10, 78A48

Key words: Convection-diffusion-reaction equation, dual reciprocity method, radial basis functions, fundamental solution.

1 Introduction

Combining of two dimensional (2D) steady state convection equation and diffusion equation with a linear reaction term leads to the 2D steady state convection-diffusion-reaction (SSCDR) equation. Many branches of science and engineering, especially in transport and fluid dynamics problems, governed by these type of equations and their combinations [1–5]. Modeling chemical, biochemical, physical, environmental and biological phenomena mathematically are samples of these transport problems.

Some researchers attempted to solve destabilizing effects caused by the presence of the first order derivatives in the CDR equations. Authors used different methods for the problem such as finite difference, finite volume and especially various types of finite element methods as well as combinations of them [5–16]. Today, we are still required better numerical technique for more accurate approximating partial differential equations

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(PDEs) that include first order spatial derivatives (convective term) and hence it is an attractive and challenging issue in fluid dynamics problems [17–19].

The aforementioned numerical methods have been employed as computational tools for discretization in computational fluid dynamics. The boundary element method (BEM) is another technique [20–24] for solving efficiently different types of PDEs such as linear and non-linear, homogeneous and non-homogeneous, time-dependent and steady state equations in different engineering problems. Bozkaya and Tezer-Sezgin [25] developed a BEM formulation for the solution of magneto-hydrodynamic flow in a semi-infinite duct. Hosseinzadeh et al. [26] recently utilized the constant and continuous linear BEMs to obtain the numerical solution of the coupled equations in velocity and induced magnetic field for the steady magneto-hydrodynamic (MHD) flow through a pipe of rectangular and circular sections having arbitrary conducting walls. Damanpack et al. [27] applied BEM to the bending analysis of thin functionally graded plates. Pisa and Aliabadi analysed bond-line cracks in thin walled aircraft structures by using the BEM [28].

The main advantage of this approach is to transform the domain integrals arising from PDEs into the boundary integrals by using the Green's second identity and a fundamental solution, since it reduces the dimension of the problem by one unit and then saves computational time and memory. However the main disadvantages of the BEM application to non-homogeneous and more general PDEs are due to the facts that the explicit fundamental solutions are not available and thus additional internal domain discretization is needed [23, 24, 29].

The DRBEM which was originally introduced by Nardini and Brebbia [30] is thus the most successful method for keeping above advantages. The DRBEM rearranges the PDE terms into an equation to find the fundamental solutions and then approximates the nonhomogeneous term b by interpolating functions $f(r)$, called radial basis functions (RBFs) in terms of its values at the collocation points, where r is the distance between a source point and the field point. Dehghan and Mirzaei [31, 32] have applied the boundary integral equation and DRBEM for solving 1D Cahn-Hilliard equation and 2D sine-Gordon equation respectively. Yun and Ang [33] treated the DRBEM for axisymmetric thermo-elastostatic analysis of non-homogeneous materials. In [34] a numerical method based on the boundary integral equation and dual reciprocity method for solving the 1D advection-diffusion equations is presented. Dehghan and Ghesmati [35] applied the dual reciprocity boundary integral equation technique to solve the nonlinear Klein-Gordon equation. A DRBEM formulation utilized for the solution of incompressible magneto-hydrodynamic (MHD) flow equations by Bozkaya and Tezer-Sezgin [36]. Purbolaksono and Aliabadi employed the DRBEM to evaluate large deformations of shear deformable plates [37]. Authors in [38] employed the DRBEM to evaluate the particular solution of Poisson equation with multi-quadratic functions. Tanaka and Chen [39] presented combined application of DRBEM and differential quadrature method to time-dependent diffusion problems.

Although many works have been done regarding to the DRBEM solving the different equations but efficient numerical schemes for solving general equations are scarce. This