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Moving Mesh Finite Element Method for Unsteady Navier-Stokes Flow

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Abstract. In this paper, we use moving mesh finite element method based upon $4P_1-P_1$ element to solve the time-dependent Navier-Stokes equations in 2D. Two-layer nested meshes are used including velocity mesh and pressure mesh, and velocity mesh can be obtained by globally refining pressure mesh. We use hierarchy geometry tree to store the nested meshes. This data structure make convienence for adaptive mesh method and the construction of multigrid preconditioning. Several numerical problems are used to show the effect of moving mesh.

AMS subject classifications: 65M60

Key words: Navier-Stokes system, $4P_1-P_1$, hierarchy geometry tree, moving mesh method.

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1 Introduction

It is important that solving incompressible Navier-Stokes equations by mixed finite element method. In order to satisfy the LBB condition, one way is to enhance velocity space relative to pressure, for example Taylor-Hood element. The other is imposing constraint on pressure space, such as stablized P_1-P_1, P_1-P_0 (see [1, 2]). In practical computing, adaptive scheme is often used to decrease the computational cost for efficency. Meanwhile, it can improve the quality of solutions locally see [3]. There are some works using h-adaptive P_2-P_1 element because of simplicity, see (see [4–6]) for detail. However, if domain has some corners or solutions have some singularities, we tend to use lower order approximations in engineering computation. Adaptive method with stabilized P_1-P_1 and P_1-P_0 elements are proposed in [7]. In [8], dual element P_1-P_0 is used for moving mesh method. It has some technical difficulties in applying adaptive mesh based on unstable element pairs. So $P_1\text{iso}P_2P_1$ (see [9–11]) is considered. In [10], it is pointed out that $P_1\text{iso}P_2P_1$ satisfies the LBB condition. Four velocity elements can be obtained by refining

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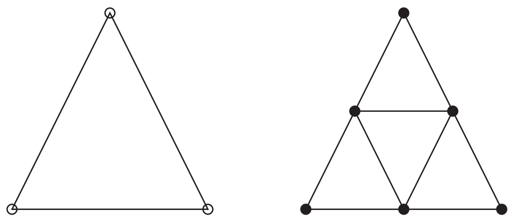

Figure 1: Left: pressure p element, $\circ$ for degrees of p; right: four velocity v elements, $\bullet$ for degrees of v.

the pressure element one time as Fig. 1 shows. Note that P_1isoP_2P_1 element is based on one set of mesh, so basis functions of pressure element are obtained by the interpolation of velocity basis functions. This will increase calculation.

In this paper, we choose $4P_1-P_1$ finite element pair. It has the same structure as P_1isoP_2P_1 pair, so the LBB condition is naturally satisfied. However, The basis functions of both velocity and pressure elements are all standard P_1 element without any extra interpolation. We use the hierarchy geometry tree which was proposed in [12] to store the mesh structure of $4P_1-P_1$ pair.

Moving mesh finite element methods have been developed by a lot of works such as [13–19]. In [16], a moving mesh finite element method based upon harmonic map was proposed. The authors in [8] extended the moving scheme to solve incompressible Navier-Stokes equations. However, the boundary conditions of numerical experiments in [8] are periodic. In this paper, we use moving mesh method based on $4P_1-P_1$ pair and the boundary conditions are general Dirichlet and Neumann boundary conditions.

The layout of the paper is arranged as follows. In Section 2, we introduce the Navier-Stokes governing equations. In Section 3, we illustrate data structure for finite element pair $4P_1-P_1$, In Section 4, $4P_1-P_1$ pair is used to approximate the governing equations. Next, the AMG preconditioner for steady stokes equations is shown. In Section 6, the moving mesh strategy is given. Then we present numerical experiments in Section 7. Finally, we give the conclusions in this section.

2 Governing equation

Following [20], we give non-dimensional incompressible Navier-Stokes equations as follows:

$$\partial_t \mathbf{u} - \frac{1}{Re}\nabla^2\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = \mathbf{f}, \tag{2.1a}$$

$$\nabla\cdot\mathbf{u} = 0. \tag{2.1b}$$

The variable $\mathbf{u}$ is velocity and the scalar variable p is pressure. The physical domain is Ω. Reynolds number $Re := \frac{UL}{\nu}$, where L represents a characteristic length scale for Ω, U is the mean velocity of the inflow and $\nu > 0$ is the constant kinematic viscosity. The