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The Sensitivity Analysis for the Flow Past Obstacles Problem with Respect to the Reynolds Number

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Abstract. In this paper, numerical sensitivity analysis with respect to the Reynolds number for the flow past obstacle problem is presented. To carry out such analysis, at each time step, we need to solve the incompressible Navier-Stokes equations on irregular domains twice, one for the primary variables; the other is for the sensitivity variables with homogeneous boundary conditions. The Navier-Stokes solver is the augmented immersed interface method for Navier-Stokes equations on irregular domains. One of the most important contribution of this paper is that our analysis can predict the critical Reynolds number at which the vortex shading begins to develop in the wake of the obstacle. Some interesting experiments are shown to illustrate how the critical Reynolds number varies with different geometric settings.

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Key words: Navier-Stokes equations, sensitivity analysis, flow past cylinder, embedding technique, immersed interface method, irregular domain, augmented system, projection method, fluid-solid interaction.

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1 Introduction

In [8], an augmented immersed interface method was proposed for the following non-dimensional incompressible Navier-Stokes equations on an irregular domain $R \setminus \Omega$:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla p = \frac{1}{Re}\Delta \mathbf{u}, \qquad \mathbf{x} \in R \setminus \Omega, \tag{1.1a}$$

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$$\nabla \cdot \mathbf{u} = 0, \qquad \mathbf{x} \in R \setminus \Omega, \tag{1.1b}$$

$$\mathbf{u}\big|_{\partial\Omega} = \mathbf{0}, \tag{1.1c}$$

$$\mathbf{u}(\mathbf{x}, 0) = \mathbf{u}_0, \qquad \text{IC}, \tag{1.1d}$$

with a usual flow boundary condition along ∂R, where R is a rectangular domain, see Fig. 1 for an illustration. In the above Navier-Stokes equations, $Re = 1/\mu$ is the Reynolds number and μ is the viscosity, $\mathbf{u}$ is the velocity, and Ω is a set of inclusions (obstacles), for example, one or two cylinders in this paper. We refer the readers to [8] and the references therein for detailed information about the problem and the related references. The method proposed in [8] is based on the augmented immersed interface method, see e.g., [9–13].

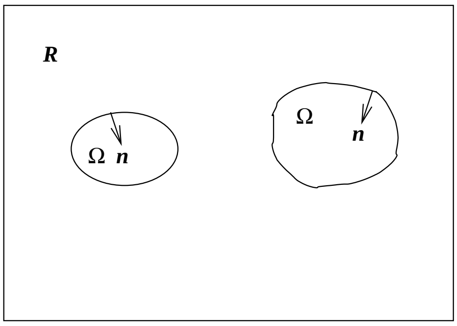

Figure 1: A diagram for the Navier-Stokes equations on an irregular domain.

In the literature, there are numerous papers on flow past cylinder problems. There are also a few of them that discuss the important quantities such as lift and drag coefficients (or forces), frequency of the vortex shading etc., as functions of the Reynolds number. In this paper, we are more interested in the sensitivity analysis with respect to the Reynolds number, or precisely $Re = 1/\mu$ when the obstacle is a circle. We believe that this paper is the first attempt to study the sensitivity of such a problem. Sensitivity analysis is important to understand how particular parameter(s) affect the solution behavior such as the stability of the flow, vortex shading behind the wake of the obstacles. As pointed out in [3], sensitivity analysis and uncertainty analysis combine to produce a systematic approach to develop a comprehensive understanding of a mathematical model, the data it produces, and the way that the data is used to influence the design of many engineering systems.

1.1 The sensitivity equations

In order to carry out the sensitivity analysis, we solve the sensitivity equations, see for example, [3, 6] and the reference therein:

$$\frac{\partial \mathbf{w}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{w} + (\mathbf{w} \cdot \nabla)\mathbf{u} + \nabla q = \frac{1}{Re}\Delta\mathbf{w} - \frac{1}{Re^2}\Delta\mathbf{u}, \qquad \mathbf{x} \in R \setminus \Omega, \tag{1.2a}$$

$$\nabla \cdot \mathbf{w} = 0, \qquad \mathbf{x} \in R \setminus \Omega, \tag{1.2b}$$