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A New Size-Dependent Cylindrical Shell Element Based on Modified Couple Stress Theory

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Abstract. In this paper, using modified couple stress theory, a new cylindrical shell element is introduced. Here, using modified couple stress theory in place of classical continuum theory, and using shell model in place of beam model which has been used in previous research, vibrational behavior and buckling behavior of nanotubes is investigated via the finite element method. In addition, the new cylindrical shell element is defined, the mass-stiffness matrix is developed, and the use of the super element and size-dependent finite element formulation together with shell element is extended to more precisely account for nanotube vibration and buckling. The new cylindrical shell element has been developed based on super element's shape function. In special cases, in order to investigate the application of the equations developed, the cylindrical nano-shell bending displacement is studied using modified couple stress cylindrical shell element and the results are validated using the analytical method. In addition, the effects of parameters such as length scale parameter, length, and thickness on cylindrical shell displacement are investigated. The results have indicated using the super element considerably reduces the amount of computations and analysis time in comparison with the use of molecular dynamic simulation.

AMS subject classifications: 70J30, 74K25

Key words: Modified couple stress theory, shell super element, size dependent, shell model, FEM.

1 Introduction

With the progress of nano-science, new micro and nano scale devices whose efficient mechanical, chemical, and electronic features have resulted in their extensive application in nano/micro structures such as micro-electromechanical systems (MEMS), nano-electromechanical systems (NEMS), and atomic force microscopes (AFMS) have attracted

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the attention of many researchers. In all these applications, according to experimental results, size effects play a significant role in proper study of the behavior of such structures. Therefore, since classical continuum theories, due to their lack of intrinsic length scales, are unable to correctly predict the behavior of these structures, use of higher order theories, such as strain gradient theory and couple stress theory, which are able to account for size effects in computations, has become popular [1–5].

In the 1990s, a number of researchers such as Koiter, Mindlin and Tiersten, and Toupin developed the couple stress theory by considering higher order rotation gradient as a deformation matrix excluding the symmetric part of the second order deformation gradient [6–9]. Using the couple stress theory, Zhou et al. investigated the mechanical and static behavior of a microbar subjected to torsional loading [10]. Also, using couple stress theory, Kang et al. investigated the micro-beam resonant frequency, demonstrating the dependency of natural frequency on size effects [11]. Following the development of this theory which includes two higher order material length scale parameters as well as two lame constants, Yang et al. developed the modified couple stress theory which includes only one material length scale parameter [12]. This theory introduces a new equilibrium equation, i.e., the equilibrium of moments of couples, in addition to equations of equilibrium and momentum of forces. Many studies have employed modified couple stress theory. For instance, using Timoshenko beam model and modified couple stress theory, Ma et al. investigated the formulation of axial and transverse deformation of microstructures [13]. Also, using this theory, Kong et al. investigated the mechanical issues of an Euler-Bernoulli beam. They evaluated the effects of size on natural frequencies under two types of boundary conditions, demonstrating a major inconsistency between the results obtained by modified couples stress theory and those by classical continuum theory in cases where the ratio of size to internal material length scale parameter characteristic was approximately equal to 1 [14]. In another study using modified couple stress theory, Reddy examined the bending of axisymmetric circular plates, demonstrating the possibility of use of the equations developed for extending the analytical response to free vibrations, bending, and buckling of linear cases [15].

In order to correctly predict the behavior of micro/nanostructures, in addition to considering the length scale parameter, it is also necessary to use an appropriate geometric model to correctly model structures and elements. This has become especially important because nanotubes have extensive application in nanostructures nowadays [16–19]. To date, many studies have been conducted using modified couple stress theory and Timoshenko and Euler-Bernoulli beam models [20–23]. For example, using Euler-Bernoulli beam model and modified couple stress theory, Akgz et al. examined the vibration frequency of non-homogenous and non-uniform micro-beams [24]. Mohammad-Abadi et al. studied the buckling of three micro-beam models, i.e., Euler-Bernoulli, Timoshenko, and Reddy beam models, using modified couple stress theory, demonstrating the effect of beam thickness, material length scale parameter, and Poisson's ratio on the critical buckling load of micro-beams [25]. Using von Karman geometrically nonlinear theory as well as Euler-Bernoulli beam model and modified couple stress theory, Wang et al. in-