Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **10**, No. 4, pp. 948-977

DOI: 10.4208/aamm.OA-2017-0220
August 2018

Solitary Wave and Quasi-Periodic Wave Solutions to a (3+1)-Dimensional Generalized Calogero-Bogoyavlenskii-Schiff Equation

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Received 1 September 2017; Accepted (in revised version) 22 December 2017

Abstract. A (3+1)-dimensional generalized Calogero-Bogoyavlenskii-Schiff equation is considered, which can be used to describe many nonlinear phenomena in plasma physics. By virtue of binary Bell polynomials, a bilinear representation of the equation is succinctly presented. Based on its bilinear formalism, we construct soliton solutions and Riemann theta function periodic wave solutions. The relationships between the soliton solutions and the periodic wave solutions are strictly established and the asymptotic behaviors of the Riemann theta function periodic wave solutions are analyzed with a detailed proof.

AMS subject classifications: 35Q51,35Q53, 35C99, 68W30, 74J35

Key words: A (3+1)-dimensional generalized Calogero-Bogoyavlenskii-Schiff equation, Bell polynomial, solitary wave solution, periodic wave solution, asymptotic behavior.

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1 Introduction

It is well known that the study on exact solutions of nonlinear evolution equations (NLEEs) is always one of the central themes in fluids, fiber optics and other fields [1]. Recently, there has been paying more attention to some generalized NLEEs because of their wide range of applications in various physical fields. It is of importance and practical significance to systematically investigate integrable properties and various exact analytic solutions to those NLEEs, both constant-coefficient and variable-coefficient. In the past decades, different solution methods have been developed in a variety of directions. Various kinds of exact solutions such as solitons, cuspon, positons complexitons and quasi-periodic solutions have been presented for NLEEs. Available solution methods include the inverse scattering transformation [1], the Hirota direct method [2], Lie group method [3], Darboux transformation and Bäcklund transformation [4,5] and the algebro-geometrical approach [6]. The Hirota direct method is one of the most powerful analytic tools for solving soliton problems of NLEEs. If a bilinear representation is known for a given NLEE, one can find its soliton solutions, bilinear BT and some other integrable properties [7–9] directly.

Based on the Bell polynomials, the Hirota bilinear method has also been developed to obtain explicit periodic wave solutions based on the Riemann theta functions. In 1980s, Nakamura proposed a direct method to construct a kind of quasi-periodic wave solutions for nonlinear equations in his essay [10], where the periodic wave solutions of the KdV equation and the Boussinesq equation were obtained by means of the Hirota direct method. The presented method only depends on the existence of Hirota bilinear forms, rather than relies on the Lax pairs and their induced Riemann surfaces for the considered equations. Recently, this method has been extended to investigate the discrete Toda lattice, $(2+1)$-dimensional Bogoyavlenskiss's breaking soliton equation and the asymmetrical Nizhnik-Novikov-Veselov equation by Fan and Hon [11–14]. One of the authors (Ma) constructed one- and two- periodic wave solutions for a class of $(2+1)$-dimensional Hirota bilinear equations and a class of trilinear differential operators used to create trilinear differential equations [15–19]. Zhang et al. [20] constructed periodic wave solutions of the Boussinesq equation. Chen et al. [21, 22] obtained a Maple package to construct bilinear forms, bilinear Bäcklund transformations, Lax pairs and conservation laws for Korteweg-de Vries-type equations. One of our authors (Tian) and his collaborators [23–27] presented soliton solutions, Riemann theta function periodic wave solutions and integrabilities of some nonlinear differential equations, discrete soliton equations and supersymmetric equations, etc.

In this paper, we focus on a (3+1)-dimensional generalized Calogero-Bogoyavlenskii-Schiff (gCBS) equation

$$u_t-h_1(uu_y+u_xv)-h_2uu_z-h_3u_{xxy}-h_4u_{xxz}+h_5u_x+h_6v_y+h_7w_z-h_8u_xw=0, \tag{1.1a}$$

$$u_y=v_x, \tag{1.1b}$$

$$u_z=w_x, \tag{1.1c}$$