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Modelling and Numerical Valuation of Power Derivatives in Energy Markets

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Abstract. In this work we investigate the pricing of swing options in a model where the underlying asset follows a jump diffusion process. We focus on the derivation of the partial integro-differential equation (PIDE) which will be applied to swing contracts and construct a novel pay-off function from a tree-based pay-off matrix that can be used as initial condition in the PIDE formulation. For valuing swing type derivatives we develop a theta implicit-explicit finite difference scheme to discretize the PIDE using a Gaussian quadrature method for the integral part. Based on known results for the classical theta-method the existence and uniqueness of solution to the new implicit-explicit finite difference method is proven. Various numerical examples illustrate the usability of the proposed method and allow us to analyse the sensitivity of swing options with respect to model parameters. In particular the effects of number of exercise rights, jump intensities and dividend yields will be investigated in depth.

AMS subject classifications: 65M10, 91B25

Key words: Swing options, jump-diffusion process, mean-reverting, Black-Scholes equation, energy market, partial integro-differential equation, theta-method, Implicit-Explicit-Scheme.

1 Introduction

In recent years the deregulation process has been developed strongly in the power market in many western countries. Although the establishment of power exchanges was not inevitable, but with its existence, markets gained more transparency and competitiveness. The European Energy Exchange (EEX), Amsterdam Power Exchange

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(APX), Nord Pool with Eltermin and Eloption and New York Mercantile Exchange (NYMEX) are the results of the emergence of markets for exchange and trading power.

In most deregulated markets the spot prices do not vary with the season only, one also observes occasional distinctive *price spikes* which are caused when the maximum supply is close to or lower than the current demand. Such supply congestion occurs when a generator or part of the distribution network fails unexpectedly or sudden change in demand could lead to spikes. Also, transfer of natural gas through pipelines and electricity through transmission lines create additional *volume constraints*.

The rise of trading electricity delivery contracts aims to satisfy the power requirement on one hand and on the other hand to minimize the new price risks. One of the first types of contracts to be traded on exchange were *futures contracts* which give the holder the obligation to buy a fixed amount of the commodity for a predefined price at a predetermined time. With many advantages, such as low transaction costs, guaranteed physical delivery and financial performance by the exchange, futures contracts are quite popular in power markets. However, since the owner of a futures contract had to use the ordered amount up or any surplus energy was wasted, this standard contract is not sufficient for market participants who find it difficult to predefine their future need of the commodity, especially when the relevant product cannot be stored or storage is very expensive. Among this immense variety of contracts *swing options* are the most suitable in hedging extreme price fluctuations of certain commodities. In allowing *flexibility-of-delivery* with respect to both timing and the amount of energy used, swing contracts can be found in coal markets, gas markets and are especially favored in energy markets, since energy is only storable in a limited manner and its storage process is quite costly e.g., in a pumped-storage power station.

From a mathematical point of view, the pricing of these power derivatives contains several challenges. One is the choice of an appropriate stochastic process which describes the price movement appropriately as electricity spot prices are mean-reverting, exhibit strong seasonalities and jump behaviour. Typically, the spot price, after some sudden jumps, immediately reverts to the long term price level. Another critical aspect is the application of the no-arbitrage approach from financial mathematics presuming markets are liquid meaning that the underlying asset can be bought and sold at a reasonable price at any time. Due to the non-storability and non-trading character of electricity and gas the no-arbitrage approach in financial market can not be applied on power markets without hurdles as buying the underlying today and selling the product in the future is not feasible. However, since the classical arbitrage pricing incorporates many advantages, many authors have been dealing with the question: How can we use arbitrage pricing in incomplete markets?

Instead of modeling the electricity market directly and deriving the forward price, one approach is to model the forward price directly as forward contracts are tradable and hence the no-arbitrage valuation can be applied. Unfortunately, forward contracts are traded only in certain time periods and hence, the liquidity is rather low. For example, the scope of delivery in EEX forward markets starts by a month, a quarter, a year and ends at three years. Thus these forward models do not reflect the price