Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **4**, No. 3, pp. 354-364

DOI: 10.4208/aamm.10-m11156
June 2012

A Constitutive Model for the Creep Behavior of Offwhite Marbles

X. D. Song[1,*] and J. H. Ren[2]

[1] *College of Science, Yanshan University, Qinhuangdao 066004, China*
[2] *College of Civil Engineering & Mechanics, Yanshan University, Qinhuangdao 066004, China*

Received 5 October 2011; Accepted (in revised version) 9 November 2011

Available online 30 April 2012

Abstract. This paper reports an improved constitutive model for the shear creep behavior of offwhite marbles which are selected from slope and underground cavern and contain green schist's weak structural planes. The shear creep behavior of the samples is characterized using the rheological tests. Based on the experimental measurements on mechanical properties under different normal stress conditions, an improved model is proposed to analyze the experimental results. It is demonstrated from a further discussion that such model can reflect the non-linear creep characteristics of structural planes, and especially, it is suitable for description of the viscoelastic and viscoplastic deformation behavior of structural planes.

AMS subject classifications: 65Z04, 76T05

Key words: Rock structural plane, shear creep, creep curve, creep model.

1 Introduction

The structural plane has a large influence on the rock heterogeneity, anisotropy and mechanical properties, other characteristics such as strength, deformation, failure and creep will later influence rock's actual state, construction, transportation, stability and reinforcement work. Rheological characteristic of rock mass discontinuity is one of the most important mechanical characteristics of rock mass. It controls the creep transfiguration and long-term strength. However, the structural plane's shear creep is complicated and has many influencing factors, on which few researches focus on. Only Amadei and Ding [1–5] carried out studies on structural plane model and created experimental and component models. The current research on structural planes under shear creep conditions [6–15] still lacks deep and systematic study; there is a need of

*Corresponding author.
URL: http://lxy.ysu.edu.cn/lxy/tec-detail.php?id=15&way=list
Email: songxd@ysu.edu.cn (X. D. Song), jhren@ysu.edu.cn (J. H. Ren)

deeper and more comprehensive studies. Here it should be indicated that under normal circumstances, marble does not show significant rheological behavior. However, in the present case, the marble with green schist's weak structural planes can exhibit an obvious time effect due to the deep and high stress environment and hence should result in a large rheological deformation. It is therefore reasonable and important to conduct the creep tests for understanding the mechanical behavior.

This paper mainly focused on the improved model which can reflect the non-linear creep characteristics of structural planes, and describe the viscoelastic and viscoplastic deformations of structural planes. The marble samples with weak structural plane will be tested through multi-stage loading shear rheology tests by rock rheology biaxial testing machine CSS-1950. The test results will be used to evaluate the creep behavior of weak structural plane and the improved model will be used to modify and analyze the results.

2 Experimental measurements

The test specimen is a $10 \times 10 \times 10\text{cm}^3$ cubic sample and the samples are selected from slope and underground cavern which contains green schist's weak structural plane. In order to maintain the quality, infrared ray is used during the process to make prospective cutting. The sample flatness is therefore fully guaranteed. The shear creep tests of green schist's weak structural plane are under 5 types of normal stresses: 5Mpa, 7.5Mpa, 10Mpa, 12.5Mpa and 15Mpa. The creep curve for the whole test process is in Fig. 1.

From the curve of the whole creep test process, it's been discovered that at higher level shear stress stages, the creep deformation clearly increases. Under very high stresses, accelerated creep stage occurs (Fig. 2), the relationship between creep rate and time is like a basin-shaped curve, see Fig. 3, the creep rate is initially large, gradually decreases, and eventually stabilizes, then later goes through rapid increasing process,

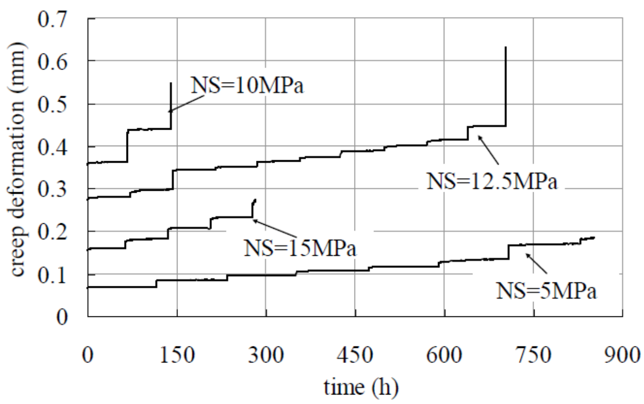


Figure 1: Creep curves of the structural plane under 4 types of normal stresses of 5Mpa, 10Mpa, 12.5Mpa and 15Mpa respectively.

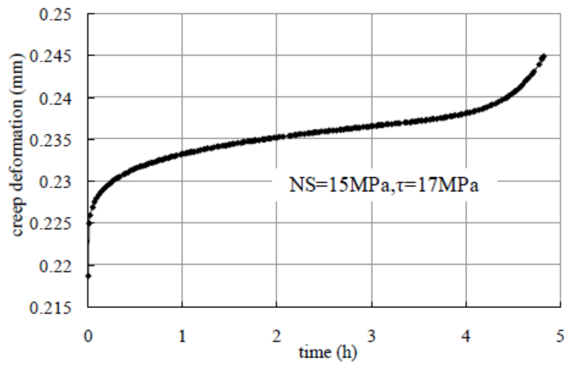


Figure 2: The structural plane accelerated creep stage curves under 15MPa stress.

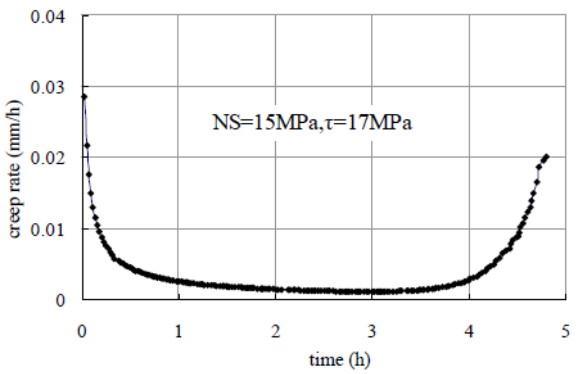


Figure 3: The structural plane creep rate curves under 15MPa normal stress.

the attenuation creep stage and the constant creep stage has the least volatile strain rate, and show a certain degree of regularity.

During the accelerated creep stage of greenschist weak structure, structural plane creep rate increases as time goes by. The power function of the creep equation (2.1) can be used to illustrate it

$$\dot{u} = \dot{u}_0 + A(t - t_c)^n. \tag{2.1}$$

In the formula, t_c is the time when the steady creep ends, $\dot{u}_0$ is the creep rate of the steady creep stage. According to the regression fitting of the experimental data, as Fig. 4 illustrates, there is a higher degree of curve fitting, describe the creep rate by using accelerated power function can be more satisfactory.

3 Theoretical considerations

From the past research results, it can be seen that Burgers model and broad sense Kelvin model can better describe the instantaneous deformation, initial creep, steady creep, as well as accelerating creep that happen in shear creep of rock structural plane.

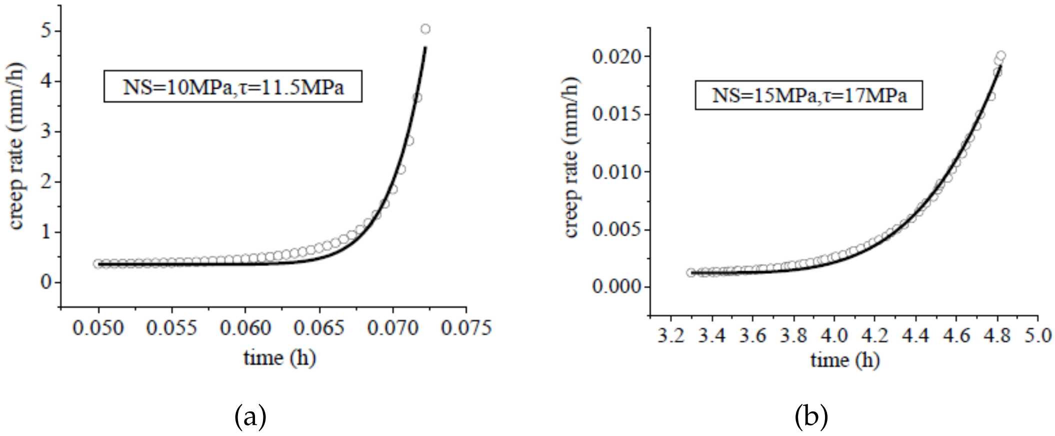


Figure 4: The accelerated creep stage and creep rate curves of the Greenschist weak structural plane.

But these two models can not describe the failure behaviors of structural plane. In order to describe the whole creep process of structural plane (instantaneous deformation, initial creep, steady creep, accelerating creep), non-linear creep model should be used.

Under grading loading conditions, the shear creep deformation of weak structural plane can be classified into elastic deformation γ_e, viscoelastic deformation γ_{ve} and viscoplastic deformation γ_{vp}:

$$\gamma = \gamma_e + \gamma_{ve} + \gamma_{vp}. \tag{3.1}$$

According to the creep deformation structure of this test, Nishihara model with five elements can be used to describe attenuation creep and steady creep stages. Nishihara model is constituted by broad sense Kelvin body and a viscous body series, see Fig. 5, Nishihara model can fully describe attenuation creep, steady creep, as well as non-steady creep.

In Fig. 8, there is no viscoplastic model when $\tau \leq \tau_s$, thus this model degraded into Kelvin model (Formula (3.2b)), at this moment, the creep constitutive equation of

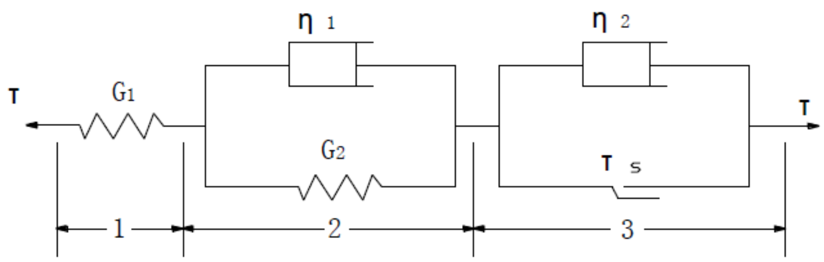


Figure 5: Component picture of Nishihara model.

Nishihara model is given below when $\tau > \tau_s$:

$$\gamma = \gamma_e + \gamma_{ve} = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big(1 - \exp\Big(-\frac{G_2}{\eta_2}t\Big)\Big), \tag{3.2a}$$

$$\gamma = \gamma_e + \gamma_{ve} + \gamma_{vp} = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big(1 - \exp\Big(-\frac{G_2}{\eta_2}t\Big)\Big) + \frac{\langle \tau - \tau_s \rangle}{\eta_1}t. \tag{3.2b}$$

Formula (3.3) and Burgers model are similar in form, but different in essence. Burgers model has only viscous deformation, no viscoplastic deformation, but Nishihara model is in the opposite side. When Nishihara model enters the stage of viscoplastic deformation, it almost has the same result as Burgers model when describing non-zero rate creep deformation. Burgers model can describe attenuation creep and viscous steady creep, but without yield limit, it cannot describe creep regularities above the long-term strength, in this model, permanent deformation is resulted by pure viscous flow. Nishihara model can describe not only attenuation creep of structural plane below long-term strength, but also steady creep above long-term strength. This model is constituted by linear rheological elements series, thus it can only reflects linear viscoelastic properties, and no accelerating properties.

From the shear creep test result of structural plane, we can know the reason why non-linear shear rheology appears at green schist weak structural plane: under constant shear stress conditions, the microcracks in rocks will experience a revolutionary process of extended fracture, thus it can be concluded that non-linear shear creep deformation is a function of time.

This part of thesis will analyze the curves of the whole creep process, as well as the strain rate of accelerating creep. In order to describe steady creep and accelerating creep under high stress, a new non-linear rheology element will be connected in series in Nishihara model. Considering the following factors: the non-linear increasement of creep rate at accelerating stage; viscosity coefficient decreases as time goes on; and at accelerating stage, creep rate of structural plane increases rapidly in the form of power function as time goes on, thus the relationship between time and creep deformation of non-linear rheology element is shown below:

$$\gamma = \frac{\tau}{\eta}[H(t - t_c)]^n. \tag{3.3}$$

where η and n are creep parameters, n is defined as rheology index, which is used to reflect the speed degree of accelerating creep at structural plane, t_c is the initial time of transition from steady creep to accelerating creep, H is a switching function given by

$$[H(t - t_c)] = \begin{cases} 0, & t \le t_c, \\ t - t_c, & t \ge t_c. \end{cases} \tag{3.4}$$

We can see from Formula (3.3) and Fig. 6, after $t > t_c$, shear creep and creep rate of structural plane show non-linear increasement with time, thus this non-linear element

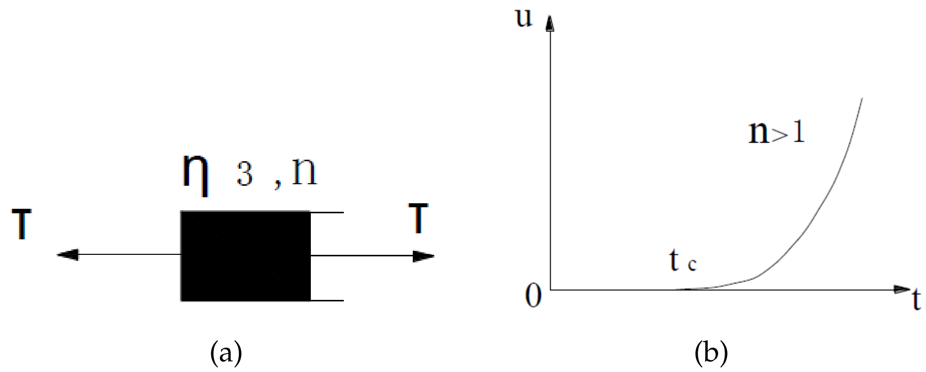


Figure 6: Diagram of (a) non-linear shear rheology element and (b) creep curves.

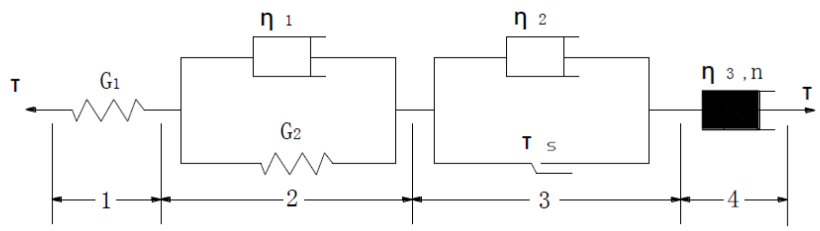


Figure 7: The non-linear viscoelastic plasticity shear rheology model of structural plane.

can fully reflect accelerating rheology properties of structural plane, and can be used to describe non-linear accelerating rheology deformation.

This shear creep test of green schist structural plane is a complicated process which consists of elastic, viscous, plastic, viscoelastic and viscoplastic deformation, therefore the combination of various elements should be applied to simulate the creep process. According to the above analysis, it can be seen that Nishihara model can better describe attenuation creep and steady creep in this creep test, on the other hand, non-linear shear rheology can better simulate the accelerating creep stage. Combining them together, that is to say, connecting Nishihara model and non-linear rheology in series, a new non-linear viscoelastic plasticity shear rheology model, which can comprehensively describe shear creep properties of green schist weak structural plane (Fig. 7), can be established.

According to the series-parallel properties of elements, the non-linear viscoelastic plasticity rheology model of structural plane consists of 4 parts. Therefore, we can get the constitutive relations under one-dimensional stress conditions:

$$\tau = \tau_1 + \tau_2 + \tau_3 + \tau_4, \qquad \gamma = \gamma_1 + \gamma_2 + \gamma_3 + \gamma_4, \tag{3.5a}$$

$$\gamma_1 = \frac{\tau_1}{G_1}, \qquad \gamma_2 = \frac{\tau_2}{G_2}\Big[1 - \exp\Big(-\frac{G_2}{\eta_1}t\Big)\Big], \tag{3.5b}$$

$$\gamma_3 = \begin{cases} 0, & \tau_3 < \tau_s, \\ \dfrac{\tau_3 - \tau_s}{\eta_2}t, & \tau_3 \geq \tau_s, \end{cases} \qquad \gamma_4 = \begin{cases} 0, & 0 < t_c, \\ \dfrac{\tau_4}{\eta_3}(t - t_c)^n, & t \geq t_c. \end{cases} \tag{3.5c}$$

In the formulae, G_1, G_2 are shear modulus, η_1, η_2, η_3 are viscosity coefficients which reflect the speed of accelerating creep on structural plane, t_c is the initial time of transition from steady creep to accelerating creep.

Based on the above formulae, the non-linear viscoelastic plasticity creep constitutive model is as follows:

(I) There exists only instantaneous deformation for the moment when shear tress is applied:

$$\gamma = \frac{\tau}{G_1}. \tag{3.6}$$

(2) After the shear tress is applied, when $\tau < \tau_s$, the structural plane doesn't enter yield situation. Under constant shear stress conditions, the creep curve only shows characteristics of attenuation creep, thus the non-linear viscoelastic plasticity creep constitutive model degrades into broad sense Kelvin model, which can describe viscoelastic properties of the structural plane:

$$\gamma = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big[1 - \exp\Big(-\frac{G_2}{\eta_1}t\Big)\Big]. \tag{3.7}$$

(3) When $\tau \geq \tau_s$, $t < t_c$, the green schist weak structural plane enter yield situation, the viscous components of the third part are involved in deformation, the model can describe attenuation creep and steady creep stages respectively. It is similar to Burger model, and can also be used to describe the viscoelastic plasticity properties of the structural plane:

$$\gamma = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big[1 - \exp\Big(-\frac{G_2}{\eta_1}t\Big)\Big] + \frac{\tau - \tau_s}{\eta_2}t. \tag{3.8}$$

(4) When $\tau \geq \tau_s$, $t \geq t_c$, the green schist weak structural plane enter accelerating creep stage, the non-linear shear rheology elements of the fourth part are involved in deformation, and the model can describe attenuation creep, steady creep, as well as accelerating stage respectively, and can describe the non-linear viscoelastic plasticity properties of the structural plane:

$$\gamma = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big[1 - \exp\Big(-\frac{G_2}{\eta_1}t\Big)\Big] + \frac{\tau - \tau_s}{\eta_2}t + \frac{\tau}{\eta_3}(t - t_c)^n. \tag{3.9}$$

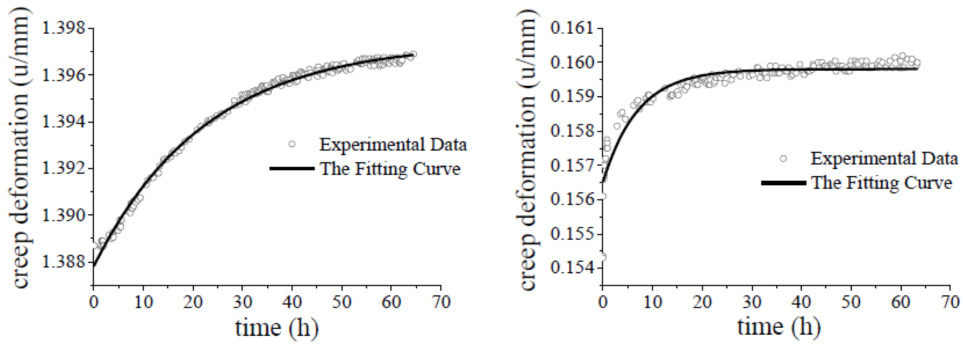
Combining Formulae (3.6)-(3.9) gives

$$\gamma = \frac{\tau}{G_1} + \frac{\tau}{G_2}\Big[1 - \exp\Big(-\frac{G_2}{\eta_1}t\Big)\Big] + \frac{\tau - \tau_s}{\eta_2}t + \frac{\tau}{\eta_3}[H(t - t_c)]^n. \tag{3.10}$$

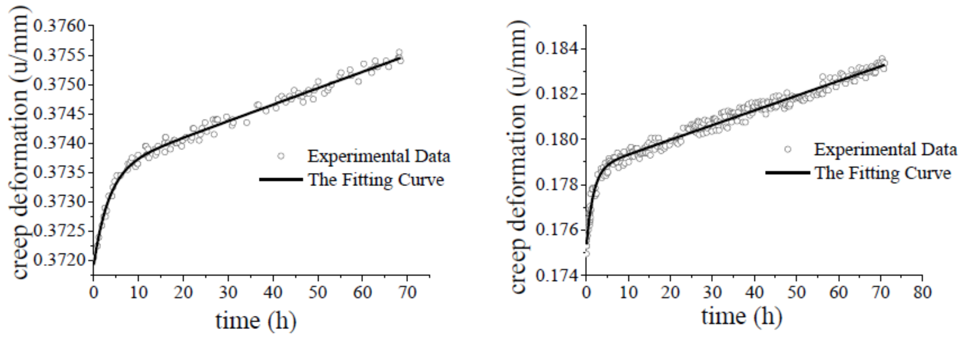
Formula (3.10) is the non-linear shear rheology model of structural plane. It can not only describe the viscoelastic plasticity properties of structural plane, but can also better reflect the non-linear accelerating creep deformation of structural plane

$$\langle \tau - \tau_s \rangle = \begin{cases} 0, & \tau < \tau_s, \\ \tau - \tau_s, & \tau \geq \tau_s, \end{cases} \qquad H(t - t_c) = \begin{cases} 0, & t < t_c, \\ t - t_c, & t \geq t_c. \end{cases} \tag{3.11}$$

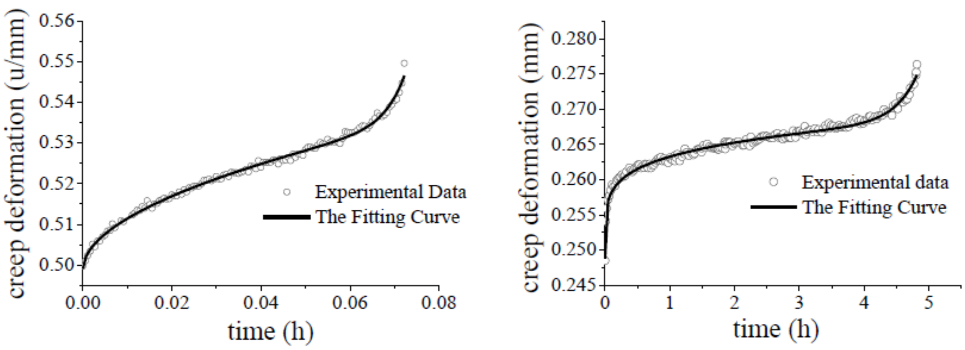
According to the long-term strength characteristics of green schist weak structural plane, the anti-shear strength of it is 60% of the quick shear strength, and on the other hand, combining the creep test curves of structural plane, we can get that the stress



(a) The greenschist weak structural plane creep decay curve fitting



(b) The greenschist weak structural plane creep curve fitting



(c) The greenschist weak structural plane accelerated creep curve fitting

Figure 8: The structural plane shear rheological model of nonlinear curve fitting.

value of structural plane is τ_s when it develops into viscoplastic deformation. Observing the accelerating creep curves of structural plane, and combining the creep rate changing curves, we can get the initial time which starts from steady shear creep to accelerating shear creep, the concrete parameters are shown in Table 1.

According to the test parameters in Table 1, and simulate the test results by using non-linear viscoelastic plasticity shear rheology model, we can get the fitting parameters of model in Table 2, and these parameters can be used to get fitting curves of

Table 1: The long-term strength parameters chart of green schist weak structural plane.

Samples No.	Σ(MPa)	τ(MPa)	τ_s (MPa)	t_c(h)
CP1	5.0	128	7.0	18.00
CP2	7.5	15.9	8.1	0.09
CP3	10.0	19.1	10.2	0.05
CP4	12.5	22.3	11.0	0.20
CP5	15.0	25.5	12.5	3.20

Table 2: The fitting parameters chart of non-linear creep model.

Samples No.	shear stresss(MPa)	G_1(GPa)	η_1(GPa)	G_2(GPa)	η_2(GPa)	η_3(GPa)	n	R
CP1	4.5	68.51	1823.25	168.25	-	-	-	0.912
	5.4	64.31	3045.71	64043.7	-	-	-	0.921
	6.3	66.77	5001.78	2092.14	164574.5	-	-	0.966
	7.2	68.72	3103.54	42904.1	286662.7	-	-	0.978
	8.1	70.95	5241.15	46717.3	700840.3	-	-	0.936
	9.0	71.16	2289.36	50107.1	93877.1	-	-	0.989
	10.2	61.14	3736.46	91554.6	385819.9	-	-	0.963
	11.5	63.83	-41887	402147	889.22	-1516.8	1	0.908
CP2	7.0	5.04	728.82	16449.3	-	-	-	0.997
	8.3	5.75	21737.9	38115.4	122921.1	-	-	0.973
	9.6	6.48	5371.66	27.668	273.4278	2.0E-11	16	0.947
CP3	7.0	19.71	1325.78	1997.98	26746.61	-	-	0.989
	9.5	19.335	1028.91	738.41	86298.61	-	-	0.988
	11.5	22.996	1166.38	6.787	12.218	8.7E-27	25	0.998
CP4	10.3	28.48	2197.78	109195	-	-	-	0.989
	11.5	30.92	7159.73	24119.1	42899.9	-	-	0.994
	12.7	33.11	3563.27	6073.22	88685.1	-	-	0.937
	13.9	35.02	2916.2	36929.7	1076421	-	-	0.998
	15.0	36.68	4375.65	24414.7	146045.7	-	-	0.981
	16.2	36.78	2875.41	1593.89	211589.6	-	-	0.977
	17.4	34.43	315.88	12.12	68.057	1.7E-28	55	0.997
CP5	9.5	60.69	2886.27	20298.1	-	-	-	0.933
	11.3	64.44	3403.82	5623.66	27842.44	-	-	0.994
	13.2	66.00	2119.20	1509.04	109859.2	-	-	0.983
	15.1	67.30	2187.18	7738.28	1065760	-	-	0.995
	17.0	68.49	-1.2E+6	1012718	217646.1	1143.13	0.2	0.989

green schist structural plane, see Fig. 8.

From the above test results, it can be concluded that the fitting curves have good fitting effects and high correlation coefficients, these all prove the appropriateness of rheology model which is based on the test mechanisms and divides creep deformation into initial creep, steady creep, as well as accelerating creep; they also prove the rationality and applicability of this model in aspects of shear creep properties. It must be explained that the above analysis proves the rationality of constructed model from the angle of indoor tests, thus the corresponding test parameters may not be used directly in field rock engineering, but they can be referential materials, to make sure the parameter scope of structural plane during back analysis of model parameters.

Finally, it is indicated that the present study focuses on the shear creep behavior of green schist's weak structural plane. The rheological tests for the strong structural planes are needed and can be compared with the present results. Nevertheless, based on the present results obtained, it can be conjectured that the strong structural planes should show obvious different rheological properties, and the most possible feature for such planes is that no accelerated creep stage could appear.

4 Conclusions

This paper employs the greenschist weak structural rock surface to investigate shear creep behavior under different normal stress conditions. The experimental results show that the creep behavior can be dived into 3 stages: attenuation, constant and accelerating. For the attenuation and accelerated stages, the structural plane displays the nonlinear characteristics, while for the stable creep stage, the deformation characteristics become more complicated. Using an improved model, the above creep characteristics of the structural planes can be described well. This improved model can reflect the non-linear creep characteristics of structural planes and describe the viscoelastic and viscoplastic deformations.

References

[1] G. N. Boukharov and M. W. Chanda, *The three processes of brittle crystalline rock creep*, Int. J. Rock. Mech. Min. Sci., 32(4) (1995), pp. 325–335.

[2] Y. Fujii and T. Kiyama, *Circumferential strain behaviour during creep tests of brittle rocks*, Int. J. Rock. Mech. Min. Sci., 6 (1999), pp. 323–337.

[3] E. Maranini and M. Brignoli, *Creep behavior of a weak rock: experimental characterization*, Int. J. Rock. Mech. Min. Sci., 36(1) (1999), pp. 127–138.

[4] X. L. Ding, L. Jian and X. Z. Liu, *Experimental study on creep behaviors of hard structural plane in TGP s permanent lock regions*, Journal of Yangtze River Scientific Research Institute, 08 (2000), pp. 30–33.

[5] P. XU and X. L. Xia, *A study on the creep model of rock mass discontinuity of the Three Gorges Project*, Journal of Yangtze River Scientific Research Institute, 03 (1992), pp. 42–45.

[6] Y. S. Li and C. C. Xia, *Time-dependent tests on intact rocks in uniaxial compression*, Int. J. Rock. Mech. Min. Sci., 37(3) (2000), pp. 467–475.

[7] S. Okubo, Y. Nishimatsu and K. Fukui, *Complete creep curves under uniaxial compression*, Int. J. Rock. Mech. Min. Sci. Geomech. Abstr, 28(1) (1991), pp. 77–82.

[8] L. Jing, E. Nordlund and O. Stephansson, *A 3D constitutive model for rock joints with anisotropic friction and stress dependency in shear stiffness*, Int. J. Rock. Mech. Min. Sci. Geomech. Abstr., 31(2) (1994), pp. 173–178.

[9] F. Yin, The Creep of Potash Rock from New Brunswick, Canada: University of Manitoba, 1998.

[10] M. L. Zhu, Z. D. Zhu and Z. J. Li etc., *Preliminary study of non-stationary shear rheological model of wall rock of long, large and deep-buried tunnel*, Chinese. J. Rock. Mech. Eng., 27(7) (2008), pp. 1436–1441.

[11] D. Sterpi and G. Gioda, *Visco-plastic behavior around advancing tunnels in squeezing rock*, Rock. Mech. Rock. Eng., 23(3) (2007), pp. 292–299.

[12] W. Y. Xu and S. Q. Yang, *Test study on shear rheological behavior of jointed rock mass and the model investigations*, Chinese. J. Rock. Mech. Eng., 24 (2005), pp. 5536–5542.

[13] T. J. H. HEEGE, J. H. P. DE BRESSER AND C. J. SPIERS, *Rheological behavior of synthetic rock salt: the interplay between water, dynamic recrystallization and deformation mechanisms*, J. Struct. Geol., 27(6) (2005), pp. 948–963.

[14] G. FABRE AND F. PELLET, *Creep and time-dependent damage in rgillaceous rocks*, Int. J. Rock. Mech. Min. Sci., 43(6) (2006), pp. 950–960.

[15] D. STERPI AND G. GIODA, *Visco-plastic behavior around advancing tunnels in squeezing rock*, Rock. Mech. Rock. Eng., 23(3) (2007), pp. 292–299.