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Contaminant Flow and Transport Simulation in Cracked Porous Media Using Locally Conservative Schemes

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Abstract. The purpose of this paper is to analyze some features of contaminant flow passing through cracked porous medium, such as the influence of fracture network on the advection and diffusion of contaminant species, the impact of adsorption on the overall transport of contaminant wastes. In order to precisely describe the whole process, we firstly build the mathematical model to simulate this problem numerically. Taking into consideration of the characteristics of contaminant flow, we employ two partial differential equations to formulate the whole problem. One is flow equation; the other is reactive transport equation. The first equation is used to describe the total flow of contaminant wastes, which is based on Darcy law. The second one will characterize the adsorption, diffusion and convection behavior of contaminant species, which describes most features of contaminant flow we are interested in. After the construction of numerical model, we apply locally conservative and compatible algorithms to solve this mathematical model. Specifically, we apply Mixed Finite Element (MFE) method to the flow equation and Discontinuous Galerkin (DG) method for the transport equation. MFE has a good convergence rate and numerical accuracy for Darcy velocity. DG is more flexible and can be used to deal with irregular meshes, as well as little numerical diffusion. With these two numerical means, we investigate the sensitivity analysis of different features of contaminant flow in our model, such as diffusion, permeability and fracture density. In particular, we study K_d values which represent the distribution of contaminant wastes between the solid and liquid phases. We also make omparisons of two different schemes and discuss the advantages of both methods.

AMS subject classifications: 65M12, 65M15, 65N30, 76D07, 76S05

Key words: Mixed finite element methods, discontinuous Galerkin methods, reactive transport equation, flow equation.

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1 Introduction

In this paper, we are interested in simulating processes in subsurface waste repositories. We have a solid concrete matrix to seal the radioactive wastes underground, however, there will be some fractures produced by the erosion of water, acid solute or other undetermined elements during the long time periods. Hence, we need to consider how these radioactive wastes leak through our concrete matrix from these apertures since the convection in fractures is much faster than that in matrix. Although this scenario is complicated to model exactly, we make reasonable simplification to create a mathematical model that can be handled numerically and yet produce accurate prediction. We shall construct a mathematical model as follows: we will consider the flow of an incompressible fluid through a homogeneous saturated porous media, where the fluid is contaminated by a solute, with concentration $c \geq 0$. We assume that the flow is at steady state and that transport is described by advection, molecular diffusion, mechanical dispersion, and chemical reaction (adsorption) between solute and the surrounding solid porous skeleton. There are lots of applications related to this model [2,9,15,16,25,30,31,33].

The contaminant flow of fluids through fractures is a process that plays an important role for many areas of the geosciences. Research on fluid flow in fractures and in fractured porous media has a history that spans nearly four decades. This research can be classified as four principal aspects of fracture flow: 1) development of conceptual models, 2) development of analytical and numerical solution schemes, 3) description of fracture hydraulic characteristics in static and deforming media, and 4) development of stochastic techniques to describe fracture flow and hydro-geologic parameter distributions. In this paper, we will firstly build the mathematical model for the original problem, then apply appropriate numerical schemes to solve this model and analyze the numerical results from our proposed methods in order to derive a better solution and accurate results.

Several conceptual models have been developed for describing contaminant fluid flow in fracture porous media. Fundamentally, each method can be distinguished on the basis of storage and flow capacities of the porous medium and the fracture. The storage characteristics are associated with porosity, and the flow characteristics are associated with permeability. There are three conceptual models which dominated the research so far: 1) dual continuum, 2) discrete fracture network, 3) single equivalent continuum. In addition, multiple-interacting continua and multi-porosity/multi-permeability conceptual models (Sahimi [24], 1995) have recently been introduced into literature. Further distinctions can be drawn on the basis of spatial and temporal scales of integration, or averaging, of the flow regime. In our research, we will mainly focus on discrete fracture network [20,21], which preserves physics closely.

After we built the mathematics model, we need to solve it with numerical algorithms. For different parts of our models, we deal with them by specific methods in order to obtain best simulation results. The movement of contaminants through the fractured porous medium is modeled by transport equations; that is, equations which