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Numerical Simulation of a Multi-Frequency Resistivity Logging-While-Drilling Tool Using a Highly Accurate and Adaptive Higher-Order Finite Element Method

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Abstract. A novel, highly efficient and accurate adaptive higher-order finite element method (hp-FEM) is used to simulate a multi-frequency resistivity logging-while-drilling (LWD) tool response in a borehole environment. Presented in this study are the vector expression of Maxwell's equations, three kinds of boundary conditions, stability weak formulation of Maxwell's equations, and automatic hp-adaptivity strategy. The new hp-FEM can select optimal refinement and calculation strategies based on the practical formation model and error estimation. Numerical experiments show that the new hp-FEM has an exponential convergence rate in terms of relative error in a user-prescribed quantity of interest against the degrees of freedom, which provides more accurate results than those obtained using the adaptive h-FEM. The numerical results illustrate the high efficiency and accuracy of the method at a given LWD tool structure and parameters in different physical models, which further confirm the accuracy of the results using the Hermes library (`http://hpfem.org/hermes`) with a multi-frequency resistivity LWD tool response in a borehole environment.

AMS subject classifications: 35Q61, 35Q86, 74S05

Key words: Resistivity logging-while-drilling, higher-order finite element method, adaptive, exponential convergence, numerical simulation.

1 Introduction

Resistivity logging-while-drilling (LWD) is an electrical logging tool that plays an im-

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portant role in oil exploration [1, 2]. At present, resistivity LWD has become a key technology in data collection in real-time well fields, interpretation and field decision-making, and guiding geology-oriented drilling [3–7]. With the extensive application of resistivity LWD, analog simulation to resistivity LWD instrument response has become a hotspot in current research, in which one of the major works is to calculate electric potential and electric field strength in different physical models. It conducts the work via the theory of a self-adaptive higher-order finite element method (*hp*-FEM), establishment of mathematical and physical models, and research on logging response using a self-adaptive *hp*-FEM [8–10], around the core of multi-frequency resistivity LWD, in the light of difficult problems in conventional resistivity logging and thin layer logging.

Based on Maxwell's equations, the current paper derives the normalized time-harmonic Maxwell's equations through the rectangular coordinate system, presents three kinds of boundary conditions used in model and stability variation equation of Maxwell's equations, and discusses the high accuracy and efficiency of the self-adaptive *hp* algorithm. In addition, the optimal *hp* refinement strategy of elements and implementation of the new *hp*-FEM are discussed, and two structural parameters used by the numerical model for numerical simulations are presented using the structure of the classical single-emission and double-receiving logging instrument. In the single-layer formation model, the relationship between electric field resistivity and signal amplitude is calculated by matching the results well with the actual situation of works, which indicates feasibility of the proposed algorithm and numerical model structure in calculating resistivity LWD response under different frequencies. In the multi-layer formation model, the effects of different frequencies on the real part, imaginary part, amplitude, phase angle, first difference value of amplitude, wall rock, and wall rock thin layer are analyzed. The resistivity LWD response curves under different frequencies are also provided.

The electric field distribution in resistivity LWD under different frequencies is simulated and analyzed using the *hp*-FEM library Hermes. The corresponding relationships between the complex media of formation and formation electric field distribution are reflected by establishing a theoretical model, which is highly significant to obtain the instrument response simulation curve, guiding design of a highly accurate resistivity LWD instrument, and logging data processing and interpretation.

2 Model

The 2D cross-section diagram of the resistivity LWD instrument in a borehole environment is shown in Fig. 1. The model is composed of a resistivity logging instrument, non-vacuum borehole, and five layers in the formation material with varying resistivities. The resistivity logging tool is used as the classical single-emission and double-receiving structure. The main parameters are as follows:

- Length of transmitting coil (0.1m);