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An Acceleration Method for Stationary Iterative Solution to Linear System of Equations

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Abstract. An acceleration scheme based on stationary iterative methods is presented for solving linear system of equations. Unlike Chebyshev semi-iterative method which requires accurate estimation of the bounds for iterative matrix eigenvalues, we use a wide range of Chebyshev-like polynomials for the accelerating process without estimating the bounds of the iterative matrix. A detailed error analysis is presented and convergence rates are obtained. Numerical experiments are carried out and comparisons with classical Jacobi and Chebyshev semi-iterative methods are provided.

AMS subject classifications: 65F10, 15A06

Key words: Iterative method, error analysis, recurrence.

1 Introduction

Linear algebraic system arises from almost every field of mathematical applications, so the problem of solving linear algebraic system is of great importance. Numerous methods have been presented for this purpose. In general, all the existing methods [1, 3, 4, 6, 9] fall into two categories: direct and iterative methods. In direct methods, one tries to decompose the coefficient matrix A in the regular system

$$Ax = b, \tag{1.1}$$

into some product form; for example in Gaussian elimination method, the coefficient

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matrix A is factored as $A = LU$, where L and U are lower and upper triangular matrices, we then solve the equivalent two simple systems:

$$Ly = b, \tag{1.2a}$$
$$Ux = y, \tag{1.2b}$$

which can be solved by using backward and forward substitution methods. By iterative method, one looks for a sequence of approximating solutions $\{x^{(k)}\}$ while the coefficients matrix A is unchanged or is just split by some simple procedures.

A large family of iteration methods for solving (1.1) take the form

$$Mx^{(k+1)} = Nx^{(k)} + b, \tag{1.3}$$

where

$$A = M - N, \tag{1.4}$$

is a splitting of the matrix A. For instance, the well-known Jacobi iteration is a member of this family with

$$M = D \quad \text{and} \quad N = -(L + U), \tag{1.5}$$

where D is the diagonal matrix with its entries exactly the same as those in A, and L and U are the lower and upper triangular matrices extracted directly from A:

$$L = (l_{ij})_{n*n} \qquad \text{with } l_{ij} = \begin{cases} 0, & i \leq j, \\ a_{ij}, & i > j, \end{cases} \tag{1.6a}$$

$$U = (u_{ij})_{n*n} \qquad \text{with } u_{ij} = \begin{cases} 0, & i \geq j, \\ a_{ij}, & i < j. \end{cases} \tag{1.6b}$$

Another example is the Gauss-Seidel iteration in which M, N are constructed as follows

$$M = D + L, \qquad N = -U. \tag{1.7}$$

The following theorem guarantees the convergence of the iteration methods defined by (1.3).

Theorem 1.1. *Suppose A, M are invertible, and the spectral radius of matrix $M^{-1}N$ is less than 1, then the iteration sequence $\{x^{(k)}\}_{k=1}^{\infty}$ produced by* (1.3) *will converge to the solution $x = A^{-1}b$ of the linear system* (1.1) *for any starting vector $x^{(0)}$.*

The above iteration methods may be attractive because of its simplicity, however the convergence of these so-called stationary methods are usually not satisfactory. Therefore some acceleration scheme is usually applied to improve the convergence of these methods. A well-known acceleration method is the Chebyshev semi-iterative method, which is discussed in [8] as well as in [7]. The following is an introduction to this method which is a variation of that in [4].